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Graph colorings and power in experimental exchange networks *

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The principal goal of studying experimental exchange networks is to understand the relationship between power and network position. In this paper we provide a formal definition of the appropriate notion of position, and explore some of the consequences of assuming that power is a function of position. It is shown that, in highly structured graphs, the space of possible power outcomes is significantly reduced if power is entirely structural. Drawing on the notion of role colorings (Everett and Borgatti 1991), we formalize the frequently expressed intuitive idea that a node's power is a function of the powers of its neighbors, just as their power is determined by the powers of their neighbors, and so on. We use a combination of two role colorings to express this idea. One, called ecological coloring, states that if two nodes have the same power neighborhoods (i.e. distinct levels of power exhibited by their neighbors), then they must have equal power. The other, called regular coloring, states that if two nodes have equal power, then we can infer that they have the same power neighborhoods. Together, these colorings imply a one-to-one relationship between the power of a node, and the power(s) of its neighbors. It is found that applying these colorings in addition to assuming power is a function of position, radically reduces the sample space of possible power outcomes, leaving only a few possibilities. With two revealing exceptions, the reduced space of possible power outcomes always contains the experimentally observed result.

In recent years, there has been considerable research investigating the structural determinants of power in experimental exchange networks (Willer and Anderson 1981, Cook *et al.* 1983; Bonacich 1987; Marsden 1987; Stolte 1988, Markovsky *et al.* 1988). The basic premise of

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this work is that social actors (in and out of the laboratory) are embedded in a network of exchange opportunities (i.e. potential trading partners), and their position in this network determines, in part, their ability to make exchanges with others at rates favorable to themselves. An actor's power is seen as a potential inherent in the actor's position that he can, if sufficiently motivated and competent, use in a well-defined exchanged situation to yield favorable outcomes. Theories accounting for power try to understand what it is about an actor's position that confers or precludes power given a specific environment of exchange rules.

In the context of laboratory experiments, these rules include such things as the number of exchanges each node is permitted per round, and whether the benefits of an exchange are conditional upon other exchanges (see the discussion on inclusive, exclusive and null connections in Willer's introduction of this issue). In this paper, we consider only experiments in which all nodes are allowed one exchange per round, and connections are not inclusive. However, our procedures apply without modification to experiments with multiple exchanges per round provided each node is allowed the same number of exchanges. In addition, we have satisfied ourselves that generalization to varying number of exchanges is possible, but is outside the scope of this paper.

1. Indices of power

One form that theories of power have taken is the development of nodal measures of *potential power* or *structural strength* based on graph-theoretic considerations (Cook *et al.* 1983; Bonacich 1987; Markovsky *et al.* 1988). Implicit in these efforts is the idea that each node in an exchange opportunity network can be assigned a score representing the strength or potential value of that location with respect to helping the actor who occupies it to exploit his or her neighbors. This strength is the result of an interaction between a specific set of rules of exchange (e.g. all actors are allowed one exchange per round), and one or more graph-theoretic properties of each location (node) in the exchange opportunity graph, such as number of alters directly tied to, proximity to nodes with few alters, etc. Thus, in an experimental setting where individual differences in competence and motivation can be controlled, once a set of rules of exchange is chosen, power relationships that emerge among actors are expected to be entirely determined by differences in strength of their locations in the experimental network [1]. We formalize these ideas in terms of two axioms, both of which assume an exchange opportunity network with a specified set of rules:

Axiom 1. Let G(V, E) be an exchange opportunity network. The *strength* $S(v)$ of a node v is a graph-theoretic attribute that can be measured on an ordinal (or higher) scale.

The term 'graph-theoretic' is used to indicate an attribute whose values are calculated from the graph itself (rather than exogenous factors such as intelligence or interest level of the actors). Examples of graph-theoretic attributes are degree, centrality, number of cliques belonged to, etc. The term 'ordinal scale' is used to indicate that strength can be expressed as numeric quantity whose value can be used to evaluate which of a pair of nodes has more of the attribute.

Axiom 2. Let G(V, E) be an exchange opportunity network. Let P be the power relation where aPb indicates a has power over b. Let S be defined as in Axiom 1. Then $\forall a, b \in V$, $aPb \langle == \rangle S(a) > S(b)$ provided $(a, b) \in E$.

According to Axiom 2, the power of an actor over another in an exchange network with a given set of rules or background conditions is wholly determined by the difference in the strength of their locations in the network. The proviso that a and b be connected is an operational requirement and not a mathematical one: since power use is defined in terms of rates of exchange, and unconnected nodes cannot exchange, power is undefined for unconnected nodes. One could further argue that if power *is* a favorable exchange rate, then power is also undefined for pairs of connected nodes that choose never to exchange. This issue is taken up again in the last section.

[1] In some experiments, however, subjects are rotated through nodes in a fractional factorial design that requires statistical control of subject effects after the data are collected. Other experiments re-use subjects in such a way that individual effects cannot be removed even statistically.

In adopting these axioms, researchers assume that, contrary to Emerson's (1962) claim, power is not, in a strict sense, truly relation. By 'relational', we mean any property, attribute or variable which describes a *pair* of nodes and which cannot be reduced to an attribute (or set of attributes) of each node individually. An example is given by a typical experimental exchange opportunity network, where links between nodes exist according to the whim of the researcher, and are not a result of any attribute of the nodes (such as propensity to exchange). The notion of testing for reducibility is analogous to testing for independence in a contingency table: if the probability of an observation falling in any cell (i, j) of the table can be computed from the probability of falling in row i and the probability of falling in column j, then we consider that there is no relationship between the variables. Similarly, if the power of one node over another is reducible to a comparison of their independent strengths, then power is no more relational than age ('is older than'), income ('makes more money than'), or any other attribute of the individual. This monadic assumption makes it not only convenient but technically correct to refer to a given node as having 'high power' or 'low power', or having 'more power than node x' rather than 'having power over x', as would be required if power were strictly relational.

It should be noted that there is nothing wrong with explaining power in terms of nodal attributes: if empirically justifiable, it is certainly parsimonious. The axioms do not imply that power is somehow non-structural nor that relations among all nodes in the network need not be consulted in developing a measure of power. The axioms do, however, imply that power is transitive (again contrary to Emerson's 1962:31 claim). That is, if a has power over b, and b has power over c, then, provided a and c are able and willing to exchange, it is inevitable that a has power over c. For clarity, we state this as a theorem:

Observation 1. Let P be a power relation on an exchange opportunity network G(V, E). Then $\forall a, b, c \in V$, aPb and bPc imply aPc, provided $(a, c) \in E$.

Proof. By the axioms, aPb and bPc imply $S(a) > S(b)$ and $S(b) > S(c)$, and since S is ordinal it follows that $S(a) > S(c)$ and therefore aPc.

Thus, conceiving of power in terms of relative structural strength has certain implications, such as transitivity, that are potentially disconfirmable. Unfortunately, none of the networks that have so far been used experimentally happen to have any triples that could be tested for transitivity [2].

2. Structural similarity

Another implication of the axioms is the fact that any pair of nodes which are structurally isomorphic must be equally strong and therefore equally powerful. By structurally isomorphic we mean that they have the exact same pattern of direct and indirect ties with others. For example, if a node a is connected to three others, one of which is connected to no other, then any node isomorphic to a must also be connected to exactly three others, one of which is connected to no other. Structurally isomorphic nodes are absolutely indistinguishable except by name. One way to identify sets of isomorphic nodes in a graph is to redraw the graph without node labels. Any nodes that can be distinguished from any other in this graph (on any ground, including centrality, number of alters at distance 2, etc.) are not structurally isomorphic. If no graph-theoretic attribute distinguishes them, then they are structural isomorphic. A technical definition is as follows:

Definition 1. Let G(V, E) be a graph with node-set V and edge-set E. Two nodes $a, b \in \mathrm{V}$ are said to be *structurally isomorphic* if there exists an automorphism π such that $\pi(a) = b$. An automorphism is a permutation of nodes such that for all $u, v \in V$, $(u, v) \in \mathrm{E}$ iff $\pi(u), \pi(v) \in \mathrm{E}$.

To illustrate the concept, consider the graph in Fig. 1(a). The set of all automorphisms of the graph is given in Table 1. As can readily be seen, the non-trivial automorphisms correspond to the vertical and horizontal symmetries of the graph, together with their composition. The maximal sets of nodes that are mapped to each other by any automorphism are $\{\{a, c, h, j\}, \{b, d, g, i\}, \{e, f\}\}$. These exhaustive,

[2] Research has been conducted almost exclusively on trees, which contain no triples in which all members are connected, as the theorem requires.

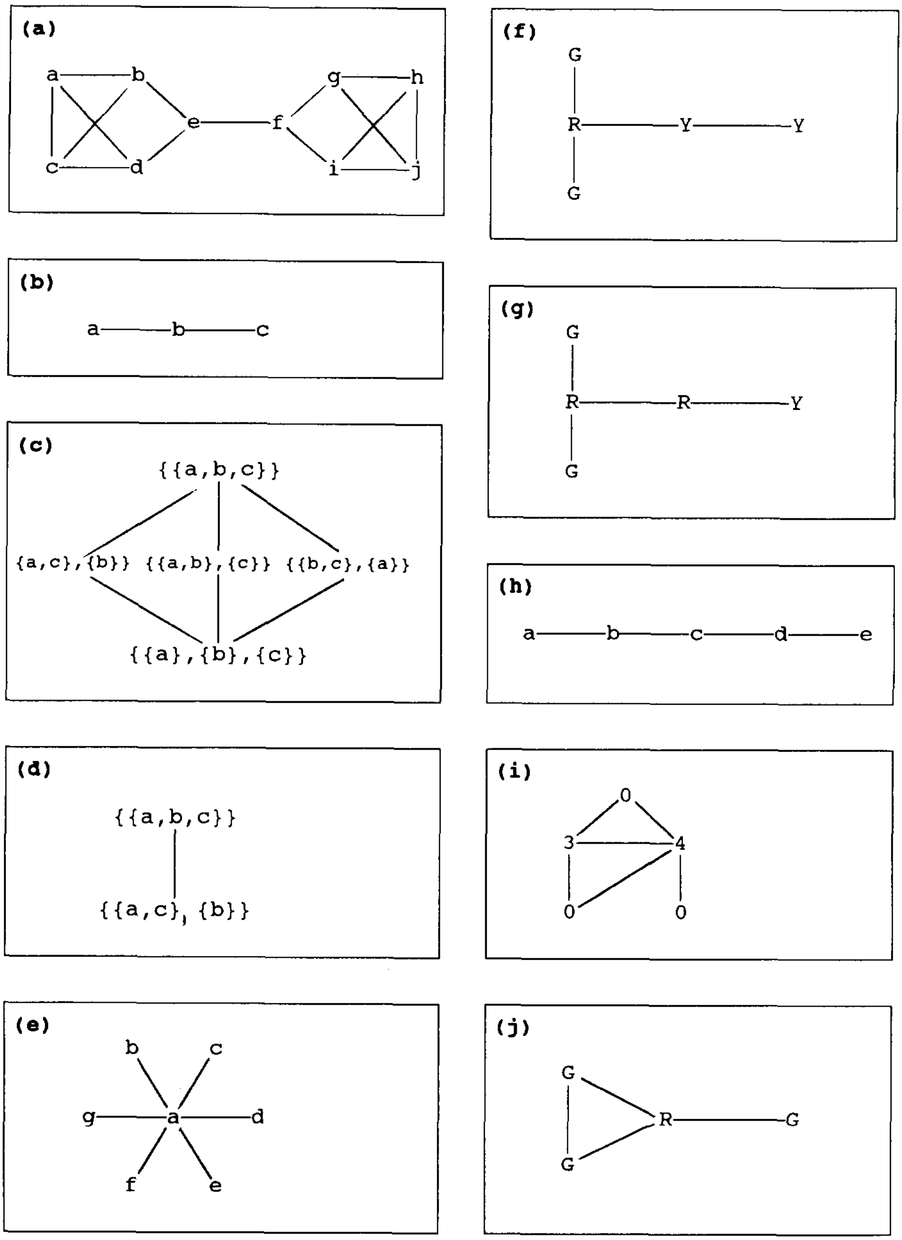

Fig. 1. Exchange opportunity networks.

Table 1
Automorphisms of the graph in Fig. 1

Node	π_1	π_2	π_3	π_4
a	c	h	j	a
b	d	g	i	b
c	a	j	h	c
d	b	i	g	d
e	e	f	f	e
f	f	e	e	f
g	i	b	d	g
h	j	a	c	h
i	g	d	b	i
j	h	c	a	j

mutually exclusive sets (called *orbits*) identify nodes that are structurally indistinguishable. Orbits and related constructions have been proposed (Everett 1985) as network formalizations of the sociological notion of *position*. For a fuller discussion of this notion of position, see Borgatti and Everett (1992).

It should be noted that the definition assumes that all nodes are allowed the same number of exchanges per round. To generalize the definition to variable numbers of exchanges, one has only to realize that two nodes allowed different number of exchanges cannot be isomorphic. In other words, the number of permissible exchanges acts like an indelible stain that disallows any automorphism that does not match colors. Thus, we can modify the definition as follows to allow variable exchanges:

Definition 1b. Let G(V, E, X) be a graph with node-set V, edge-set E and nodal attribute X such that $X(v)$ gives the number of exchanges per round that node v is allowed. Two nodes, $a, b \in V$ are said to be *structurally isomorphic* if there exists an automorphism π such that $\pi(a) = b$. An automorphism is a permutation of nodes such that for all $u, v \in V$, $(u, v) \in E$ iff $(\pi(u), \pi(v)) \in E$ and $\pi(y) = \pi(w) \rightarrow X(y) = X(w)$.

Given the axioms relating power to structural strength, it is evident that two nodes that occupy the same position must have equal strength and, if adjacent, equal power [3]. Furthermore, their power

[3] Readers may prefer to add the qualifer 'if they exchange'.

relationships vis-à-vis their respective neighbors must correspond to each other in a one-to-one fashion. Finally, the power each has over a common alter (or a structurally isomorphic alter) must be the same. These are all consequences of the following basic theorem:

Theorem 1. Let $S(v)$ be the strength of a node in an exchange network $G(V, E)$ with a given set of rules. Let $O(v)$ denote the orbit or position of node v. Then $O(a) = O(b) \rightarrow S(a) = S(b)$.

Proof. By definition, if two nodes are automorphically equivalent, they are identical with respect to all graph-theoretic attributes, including strength.

Theorem 1 simply states that structurally identical nodes must have equal strength. Note that it does not follow that nodes of equal strength are isomorphic, nor (equivalently) that nodes that occupy structurally distinct positions must have different strengths. The following corollaries are evident, given our axioms:

Corollary 2-1. If $O(a) = O(b)$ then $(a, b) \notin P$ and $(b, a) \notin P$.

Corollary 2-2. If $O(a) = O(b)$ and aPc, then $\exists d$ such that bPd and $O(c) = O(d)$.

Corollary 2-1 merely states that structurally isomorphic nodes have no power over each other. Corollary 2-2 states that structurally isomorphic nodes have isomorphic 'power neighborhoods' such that any alter that one ego dominates has a counterpart in the other ego's neighborhood which is equally dominated.

3. Power partitions

Now let us consider the space of all possible realizations of the dependent variable in an exchange experiment, which is power use. As commonly measured, the dependent variable is a ratio-scaled variable which records, for each pair of adjacent nodes, the long-run rate of exchange negotiated by the pair. The independent variable, structural strength, is also typically constructed using procedures that arguably yield ratio-scaled measurements. Until very recently, however, researchers have been quite modest in their use of these measurements, having chosen to utilize only the ordinal qualities of their measures. For example, to test their GPI measure against experimental results, Markovsky *et al.* check only whether nodes with higher GPI scores have better-than-even exchange rates with those they exchange with.

In this paper, we shall be even more modest. We shall not attempt to determine which of a pair of adjacent nodes will dominate. Rather, we shall concern ourselves only with determining whether either node will dominate, or whether they will trade as equals. In effect, we will ignore even the ordinal qualities of our measurements and make use only of the nominal.

Given that we are only interested in nominal properties, each realization of the dependent variable reduces to a binary relation which indicates, for each pair of adjacent nodes, whether they trade on equal terms. This relation is reflexive, symmetric and, in principle, transitive. It is therefore an equivalence relation which partitions nodes such that nodes placed in the same class have equal power while nodes placed in different classes have unequal power. In practice, of course, one must recognize that certain power relationships implied by the partition, namely those between non-adjacent nodes, can never be observed and are therefore best left uninterpreted.

An upper bound on the number of possible realizations of nominal power outcomes is therefore the number of partitions of n nodes. However, for many networks this is not a least upper bound since, for any graph with non-trivial automorphisms, Theorem 1 implies that some partitions are impossible. According to the theorem, isomorphic nodes must always be placed in the same power class. Hence any partition that splits isomorphic nodes into separate classes cannot be a power partition. It is well known that the set of all partitions of n objects forms a lattice. Theorem 1 says that the set of power partitions forms a subset of that lattice, with the partition of nodes into orbits as its base. This means that a power partition can differ from the orbit partition, but only by collapsing some of the orbits into broader classes.

Thus, one effect of Theorem 1 and its corollaries is to refine the sample space of possible power outcomes to a potentially much smaller set. An example is the three-node line shown in Fig. 1(b). The lattice of all partitions of three nodes (five in all) is shown in Fig. 1c.

According to Theorem 1, only two of these partitions (shown in Fig. 1(d)) are possible power outcomes. As can be seen, for graphs with a great deal of 'structure' (i.e. relatively many nodes occupying the same position), Theorem 1 provides a considerable reduction in the number of partitions that need to be considered. For a five node line, the theorem yields five possible outcomes out of 52 partitions. For a seven-node star (Fig. 1e), the theorem yields only two partitions out of 877.

Experimental support for Theorem 1 is provided by the network in Fig. 2(d), which was tested by Markovsky *et al.* Of the 877 possible partitions, only five are allowed by Theorem 1 (see Table 2). Included in the five is $\{\{a, e, f, g\}, \{b, c, d\}\}$, which is the partition empirically observed when all nodes are allowed one exchange per round. As the reader can verify, the results of all experimentally tested exchange networks reported in the literature to date are consistent with Theorem 1.

Theorem 1 may be used a a first check for proposed measures of potential power, since any measure of *positional* strength that does not assign the same values to isomorphic nodes is obviously incorrect.

It is interesting to note that the theorem applies to any structural variable, not just power. This means that for many graphs (those is which there are fewer distinct positions than nodes), there are a limited number of possible partitions that constitute the outcome space for any infinity of conceivable structural variables. For such graphs, we can expect high nominal associations among all structural attributes, including betweenness, power, eccentricity, expansiveness, closeness, etc. In general, the fewer the number of distinct positions, the higher the expected associations among all structural variables, thus changing the usual baseline. In this sense, associations among structural variables are a function of the 'amount of structure' in the

Table 2
Permissible power partitions for the graph in Fig. 2(d), given Theorem 1

1.	{a}{b, c, d}{e, f, g}}
2.	{{a, b, c, d}{e, f, g}}
3.	{{a, e, f, g}{b, c, d}}
4.	{{a}{b, c; d, e, f, g}}
5.	{{a, b, c, d, e}}

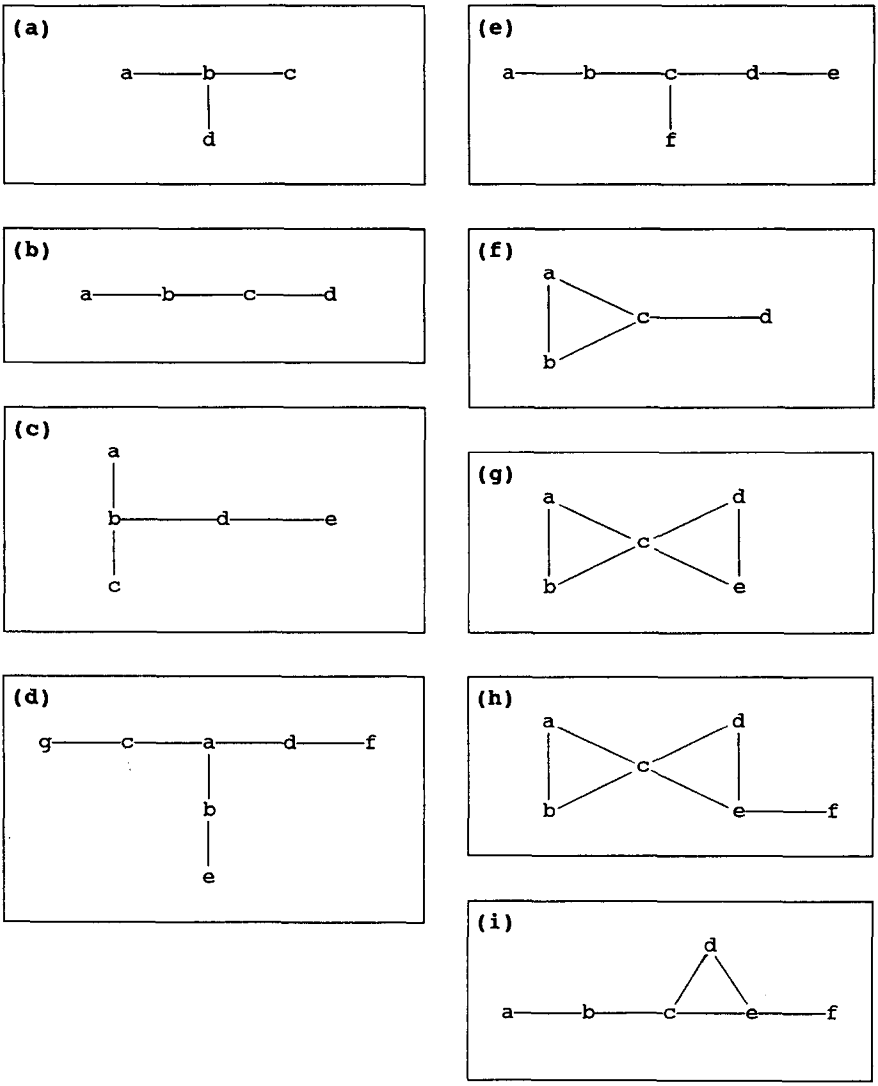

Fig. 2. Networks common to all articles in this volume.

graph, where amount of structure is taken to mean the ratio of number of nodes to number of distinct positions.

This point has some implications for power researchers. Almost every graph tested in the literature is highly symmetric with relatively many nodes relative to distinct positions. Consequently, many measures are likely to give similar results, making it difficult to distinguish among theories. In other words, graphs differ in their ability to discriminate among measures, but the field has to date chosen to test only graphs that emphasize similarities rather than differences.

At the same time, these measures are likely to be (spuriously) associated with other graph-theoretic variables such as centrality [4]. The associations are spurious if they are entirely a function of the association of each variable with a third underlying variable, namely structural similarity. Consequently, tests of hypotheses regarding relationships among measures of potential power and other graph-theoretic variables are best conducted on graphs in which every node occupies a distinct position, such as shown in Fig. 2(i). Such graphs eliminate one source of confounding association.

4. Role colorings

As Everett and Borgatti (1991) have noted, partitions of nodes into equivalence classes may be viewed as a *coloring* of the nodes. A coloring is simply an assignment of colors to nodes, where the color of a node v is denoted $C(v)$. Partitions of nodes into power classes can be discussed as a *power coloring*. Colorings that reflect a classification of nodes according to some notion of position or role are termed *role colorings*. A common characteristic of all role colorings is the idea that a node's color is related to the colors of the nodes it is adjacent to. In the language of social roles, this means that occupants of a given social position tend, ipso facto, to interact with occupants of certain types of positions, as when a doctor interacts with nurses, patients and other doctors while a teacher interacts with students, parents, administrators and other teachers. Each of these roles in turn tends to interact with a distinctive collection of roles. The partition of structurally isomorphic nodes into orbits discussed in the previous section is an example of a role coloring. In this section, we consider how power relates to two additional role colorings.

[4] Using a nominal measure of association, of course.

The reason for considering such colorings is the intuitive conviction shared by several researchers that a node's power derives from "the availability of alternative exchange relations, the unavailability of their relations' alternative relations, and so on" (Markovsky *et al.* 1988:224). Similarly, according to Cook *et al.* (1983:301), "positions are relatively 'powerless' in a network... to the extent that they have few exchange opportunities (i.e. few alternative sources of values resources) and have direct connections only to actors who have highly reliable alternative sources of supply". According to Marsden (1987:147, footnote 5), a node "may be exploitable for two reasons: It may have few alternative relations, or all of its alternatives (irrespective of how many in all are available) may be in a position to exploit others. The second condition of exploitability can lead to consideration of quite distal features of network structure".

Thus power is viewed as being determined by the power of a node's neighbors, whose powers in turn are determined by the powers of their neighbors, and so on. Such a view is clearly reminiscent of the concept of social role. We begin by defining a role concept that we shall call an *ecological* coloring:

Definition 2. Let $N(v)$ denote the set of nodes that v is adjacent to. Let $C(v)$ denote the color of a node and let $C(N(v)) = \{\cup C(u)$: $u \in N(v)$ represent the set of colors associated with v's neighbors. Then a coloring C is *ecological* if for any pair of nodes a and b, $C(a) = C(b)$ if $C(N(a)) = C(N(b))$.

According to the definition, a coloring is ecological if every node's color is entirely determined by the colors of its neighbors. Consequently, if a node u is surrounded by red and blue nodes only, and another node v is also surrounded by red and blue nodes only, then u and v must be colored the same. An example of an ecological coloring is given in Fig. 1(f). Note that every pair of nodes colored differently is surrounded by a different set of colors, as required. An example of a non-ecological coloring is given in Fig. 1(g). We hypothesize that the partition of nodes according to power forms an ecological coloring:

Proposition 1. Let P be a power coloring of an exchange network in which all nodes are allowed an equal number of exchanges. Then $\forall a, b$, $P(a) = P(b)$ if $P(N(a)) = P(N(b))$.

Table 3
Partitions consistent with Theorem 1 and Proposition 1, from the five-node line in Fig. 1(h)

	Theorem 1	Proposition 1
1.	{{a, e}{b, d}{c}}	
2.	{{a, b, d, e}{c}}	
3.	{{a, c, e}{b, d}}	{{a, c, e}{b, d}}
4.	{{a, e}{b, c, d}}	
5.	{{a, b, c, d, e}}	{{a, b, c, d, e}}

According to Proposition 1, if nodes *a* and *b* are surrounded by the same collection of powers, then *a* and *b* are themselves of equal power. Thus, if a is surrounded only by low-power nodes, and *b* is also surrounded only by low-power nodes, and *a* and *b* will be equally powerful. Note that the low-power status of *a*'s and *b*'s alters is determined by the power of their respective neighbors, whose power is determined by their neighbors, and so on. In this sense, an ecological power coloring represents an equilibrium state of the system in which all these dependencies are simultaneously satisfied.

The effect of Proposition 1, like Theorem 1, is to limit the space of possible of power outcomes. For example, for the five-node line shown in Fig. 1(h), there are five out of 52 possible partitions that are consistent with Theorem 1 (see Table 3). Of these, only two are consistent with Proposition 1. One of these is the experimentally observed partition {{a, c, e}, {b, d}}. The other is the trivial partition in which all nodes are assigned equal power. Another example is given by the graph in Fig. 2(d). Again, five partitions are consistent with Theorem 1, but only two of these are consistent with Proposition 1 (Table 4). Of these, one is the experimentally observed partition $\{\{a, e, f, g\}, \{b, c, d\}\}$, while the other is the complete partition [5].

Table 4
Power partitions consistent with Theorem 1 and Proposition 1, for the graph in Fig. 2(d)

Theorem 1	Proposition 1
{{a}{b, c, d}{e, f, g}}	
{{a, b, c, d}{e, f, g}}	
{{a, e, f, g}{b, c, d}}	{{a, e, f, g}{b, c, d}}
{{a}{b, c, d, e, f, g}}	{{a, b, c, d, e, f, g}}

Table 5
Power partitions consistent with Theorem 1 and Proposition 1, for the 'T' graph in Fig. 2(c)

Theorem 1	Proposition 1
{{a, c}{b}{d}{e}}	{{a, c}{b}{d}{e}}
{{a, c, b}{d}{e}}	
{{a, c, d}{b}{e}}	
{{a, c, e}{b}{d}}	{{a, c, e}{b}{d}}
{{b, d}{e}{a, c}}	
{{b, e}{d}{a, c}}	
{{d, e}{a, c}{b}}	{{d, e}{a, c}{b}}
{{a, c, b, d}{e}}	
{{a, c, b, e}{d}}	
{{d, e}{a, c, b}}	
{{a, c, d, e}{b}}	
{{b, e}{a, c, d}}	{{b, e}{a, c, d}}
{{b, d, e}{a, c}}	
{{e, a, c}{b, d}}	{{e, a, c}{b, d}}
{{a, b, c, d, e}}	{{a, b, c, d, e}}

Another graph that has been tested in the literature (Markovsky *et al.* 1988) is the five-node 'T' graph shown in Fig. 2(c). Fifteen partitions are consistent with Theorem 1. Of these, six are ecological, including the experimentally observed partition {{*a*, *b*}, {*b*}, {*d*, *e*}}.

It should be noted that the natural partitions induced by Markovsky *et al.*'s Graph-theoretic Power Index (where two nodes occupy the same class if and only if they have the same GPI score), are not necessarily ecological. An example is given by the graph in Fig. 1(i), in which GPI violates Proposition 1 by assigning different values to the nodes labeled *3* and *4*. However, other axioms of the original Markovsky *et al.* theory would suggest that *3* and *4* would never exchange, and therefore no difference in power can be observed.

[5] The reader is reminded that, in all cases, experimental results reported in this paper are for the case in which all nodes are permitted only one exchange per round. It is interesting to note that when nodes *a*, *b*, *c*, and d are allowed two exchanges per round, the results are consistent with the following partition {{*a*}, {*b*, *c*, *d*, *e*, *f*, *g*}}. While a formal generalization of our procedures to these kinds of conditions is outside the scope of this paper, it can be seen that the exchange opportunities facing {*e*, *f*, *g*} are quite different from those facing {*a*, *b*, *c*, *d*}. In fact, the connections among these types of points represent fundamentally different things, which are best represented by two different graphs (with different kinds of lines) on the same nodes. Since our definitions and propositions extend readily to multiple relations, this represents one path for generalization. Taking this approach leads to a set of partitions which does in fact include the experimentally observed partition.

Furthermore, in the Markovsky *et al.* formulation, the fact that one node has a GPI score of four while the other has only a three does not imply any difference in exchange rates with nodes with GPI zero. Hence, the theory as a whole would predict equal power for *3* and *4*, in accordance with Proposition 1. In fact, it can be shown that the theory is always ecologically consistent [6]. Bonacich's (1987) measure of power is also ecological in spirit, since a node's power is literally the sum of the powers of its neighbors (see Bonacich's equations 1 and 3, pp. 1172–1173).

One shortcoming of the ecological coloring is that it captures only half of the intuitive notion that power is a function of the power of a node's neighbors. In an ecological coloring two nodes with the same power neighborhoods are required to have the same power, but the converse need not be true. That is, several nodes could be assigned the same color, yet have radically different neighborhoods. An example is given by the coloring in Fig. 1(j) which assigns the same color to all peripheral nodes, and a different color to the central node. The coloring is ecological since every pairs of nodes surrounded by the same colors are colored the same. But consider the neighborhoods of the greens. Two of the greens have both a green and a red in their neighborhoods, while the other has only a red. If colors correspond to power levels, this would mean that nodes could be equally powerful, yet be surrounded by very different 'power environments'.

The converse of an ecological coloring would require that any two nodes assigned the same color have the same colors in their neighborhoods. This coloring is well known in the network role literature as *regular equivalence* (White and Reitz 1983; Borgatti and Everett 1989; Everett and Borgatti 1991). It is defined as follows:

Definition 3. A coloring C of a graph G(V, E) is regular if $\forall a, b \in V$, $C(a) = C(b)$ implies $C(N(a)) = C(N(b))$.

[6] Suppose P is a coloring consistent with Markovsky *et al.* To disconfirm the assertion, we must find a graph in which P assigns different colors (powers) to a pair of adjacent nodes who have the same colored neighborhoods. Assume node a is assigned color 1 ('high power') and node b is color 0 ('low power'). Since a and b are adjacent, $0 \in P(N(a))$ and $1 \in P(N(b))$. Then, since P is ecological there exists $c \in N(b)$ such that $P(c) = 0$. But, according to Axioms 2, 3, and 3 of Markovsky *et al.*, this would mean that a could not exert power over b since b would never seek to exchange with a. (Similarly, there must exist $d \in N(a)$ such that $P(d) = 1$. This also is forbidden by the axioms).

Table 6
Power partitions consistent with Theorem 1 and Propositions 1 and 2, for the graph in Fig. 2(f)

Theorem 1	Proposition 1	Proposition 2
{{a, b}, {c}{d}}	{{a, b}, {c}, {d}}	{{a, b}, {c}, {d}}
{{a, b, c}, {d}}		
{{a, b, d}, {c}}	{{a, b, d}, {c}}	
{{a, b}, {c, d}}	{{a, b}, {c, d}}	
{{a, b, c, d}}	{{a, b, c, d}}	{{a, b, c, d}}

To complete the formalization of the intuitive notion of power as a function of the levels of power present (or absent) in a node's neighborhood, we tentatively propose that power partitions are regular, as follows:

Propositions 2. Let P be a power coloring in an exchange network in which all nodes are allowed an equal number of exchanges. Then $\forall a, b \in V$, $P(a) = P(b)$ implies $P(N(a)) = P(N(b))$.

The effect of Proposition 2, like Theorem 1 and Proposition 1, is to further limit the space of possible of power outcomes. For example, for the four-node 'Stem' graph in Fig. 2(f), four partitions satisfy both Theorem 1 and Proposition 1. Of these, only two satisfy Propositions 2, one being $\{\{a, b\}, \{c\}, \{d\}\}$, and the other being the complete partition (see Table 6). Unfortunately, the experimental results (Markovsky *et al.* 1991) are most consistent with the partition $\{\{a, b, d\}, \{c\}\}$, which is not regular. In the experiment, c is a high power node that dominates a, b, and d. While the extent of c's observed dominance over d is not identical to its dominance over a and d, the difference is not statistically significant. Hence it is possible that a, b and d belong to the same power class, which would contradict the proposition.

A review of all other published experimental results (one exchange per round) shows that in all cases but one, the experimentally observed power partition satisfies Proposition 2. The exception (besides the 'stem' above) is the 'T' graph in Fig. 2(c), in which the experimentally observed partition $\{\{a, c\}, \{b\}, \{d, e\}\}$ is not regular (see Table 7). This suggest that either the proposition or the experimental data are wrong, or there are factors influencing power outcomes that are

Table 7
Power partitions consistent with Theorem 1 and Propositions 1 and 2, for the five-node 'T' graph in Fig. 2(c)

Theorem 1	Proposition 1	Proposition 2
{{a, c}{b}{d}{e}}	{{a, c}{b}{d}{e}}	{{a, c}{b}{d}{e}}
{{a, c, b}{d}{e}}		
{{a, c, d}{b}{e}}		
{{a, c, e}{b}{d}}	{{a, c, e}{b}{d}}	
{{b, d}{e}{a, c}}		
{{b, e}{d}{a, c}}		
{{d, e}{a, c}{b}}	{{d, e}{a, c}{b}}	
{{a, c, b, d}{e}}		
{{a, c, b, e}{d}}		
{{d, e}{a, c, b}}		
{{a, c, d, e}{b}}		
{{b, e}{a, c, d}}	{{b, e}{a, c, d}}	{{b, e}{a, c, d}}
{{b, d, e}{a, c}}		
{{e, a, c}{b, d}}	{{e, a, c}{b, d}}	{{e, a, c}{b, d}}
{{a, b, c, d, e}}	{{a, b, c, d, e}}	{{a, b, c, d, e}}

exogenous to the exchange–opportunity graph itself. We consider the latter explanation first.

One notable result of the experiments Markovsky *et al.* have conducted on this graph is that virtually no exchanges were observed between b and d – almost as if they were unconnected. If we actually did delete the link between the two and recompute the colorings (as Markovsky *et al.* do with their GPI index), the experimentally observed partition would no longer be eliminated and would, in fact, be the only non-trivial partition to satisfy both propositions and Theorem 1 (Table 8).

It is important to note that the need to delete this link is, according to Markovsky *et al.*, quite predictable, but doing so requires more information than is contained in the structure of the graph alone. It requires an assumption about the strategy and motivations of the actors, which is that actors evaluate potential trading partners, and only seek to exchange with them if they can expect to obtain a favorable deal, or if they cannot obtain a favorable deal elsewhere. Thus, certain pairs of actors, if rational, would not be expected to exchange with each other, and any link between the nodes they occupy might as well be absent. The failure of Proposition 2 when applied to the original exchange opportunity network would seem to

Table 8
Non-trivial [a] power partitions consistent with Theorem 1 and Propositions 1 and 2, for all 'common' graphs in Fig. 2

Fig.	1 Ecological	2: regular
2(a)	{a, c, d}{b}	{a, c, d}{b}
2(b)	{a, d}{b, c}	{a, d}{b, c}
2(c)	{a, c}{b}{d}{e}	{a, c}{b}{d}{e}
	{a, c, e}{b}{d}	
	{d, e}{a, c}{b}	
	{b, e}{a, c, d}	{b, e}{a, c, d}
	{e, a, c}{b, d}	{e, a, c}{b, d}
2(d)	{a, e, f, g}{b, c, d}	{a, e, f, g}{b, c, d}
2(e)	{a, e}{b, d}{c}{f}	{a, e}{b, d}{c}{f}
	{a, e}{b, d}{c, f}	
	{a, e, f}{b, d}{c}	
	{a, b, d, e}{c}{f}	
	{a, e, f}{b, c, d}	{a, e, f}{b, c, d}
	{a, c, e}{b, d, f}	{a, c, e}{b, d, f}
2(f)	{a, b}{c}{d}	{a, b}{c}{d}
	{a, b, d}{c}	
	{a, b}{c, d}	
2(g)	{a, b, d, e}{c}	{a, b, d, e}{c}
2(h)	{a, b}{c}{d}{e}{f}	{a, b}{c}{d}{e}{f}
	{a, b, d}{c}{e}{f}	
	{a, b, d}{c}{e, f}	
	{a, b}{c, f}{d}{e}	
	{a, b}{d, f}{c}{e}	
	{a, b}{e, f}{c}{d}	
	{a, b, f}{c}{d}{e}	
	{a, b, f}{c}{d, e}	
	{a b}{d, e}{c}{f}	
	{a, b}{d, e, f}{c}	
	{a, b}{d, e}{c, f}	
	{a, b}{c, e}{d, f}	
	{a, b}{c, d, f}{e}	
	{a, b}{c, d}{e}{f}	
	{a, b, c, f}{d}{e}	
	{a, b, c}{d}{e}{f}	
	{a, b, c}{d, f}{c}	
	{a, b, c, d}{e}{f}	
	{c, f}{a, b, d, e}	{c, f}{a, b, d, e}
2(i)	79 partitions	{a}{b}{c}{d}{e}{f}
		{a, f}{b, e}{c, d}
		{a, d, f}{b, 'c, e}

[a] In all cases, the partition in which all nodes have equal power satisfies both propositions and Theorem 1.

underscore the necessity of both actor-based and node-based elements in a theory of power in exchange networks, as Markovsky (1987) and Markovsky *et al* (1988) point out. The actor-based elements deal with things like strategies and motivations, while the node-based elements deal with the structure of the exchange opportunity graph (which might be the original network provided by the experimenter, but might also be a modification produced by the actors who predictably ignore certain links).

On the other hand, given the proposition's probable failure on the four-node 'stem' as well, it is also possible that this portion of our intuition about powers is simply wrong. Perhaps it is not the case that two nodes with equal power must have the same distribution of powers as neighbors. In one sense, this would not be surprising. If we are concerned with the mechanisms or processes by which laws like Proposition 1 and 2 are maintained, we have little difficulty understanding Proposition 1: a node's power is *determined* by the power of its neighbors, hence two nodes with the same distribution of powers in their neighborhoods must be equally powerful. However, the reverse is more difficult to justify. If we accept the assertion that a node's power influences the power of its neighbors, we must also accept that a given node cannot wholly determine the power of its neighbors, if they have neighbors of their own. On the other hand, the lack of an obvious mechanism is not in itself evidence against a principle. A definitive test would be a graph in which all regular colorings place a given pair of nodes in the same power class, but experimental results show a statistically significant difference in exchange rates, either with each other (if adjacent) or with equivalent others. This does not occur in any graph tested to date.

5. Conclusion

In this paper, we have sought to provide formal expression for several intuitive notions regarding the nature of power in experimental exchange networks. One fundamental notion is that power is a function of position. We formally define position in terms of graph automorphisms, then explore the consequences of this definition. For simplicity, we consider only nominal aspects of power, which is to say that we restrict all predictions to stating only whether a given pair of nodes is of equal or different power. One immediate result is that, for highly structured graphs in which many nodes occupy only a few distinct positions, the space of possible power outcomes is sharply restricted. Further, the outcome space for any structural variable, such as centrality or a proposed measure of potential power, is similarly restricted, resulting in higher than otherwise expected associations among structural variables. One lesson to be drawn from this fact is that researchers should not test theories on highly structured graphs, since these are the least likely to distinguish between competing theories. Unfortunately, most of the graphs tested in the literature to date are in fact highly structured.

Another fundamental notion about power is that a node's power is a function of the powers of its neighbors, just as their power is determined by the powers of their neighbors, and so on. We formalize this frequently expressed idea through the use of two role colorings. One, ecological coloring, states that if two nodes have the same power neighborhoods (i.e. distinct levels of power exhibited by their neighbors), then they must have equal power. The other, regular coloring, states that if two nodes have equal power, then we can infer that they have the same power neighborhoods. Together, these colorings imply a one-to-one relationship between the power of a node, and the power(s) of its neighbors. By assuming that power partitions form colorings that are both regular and ecological, we find that the sample space of possible power outcomes is even further reduced, leaving only a few possibilities. We show that both the theory of Markovsky *et al.* (1988) and the power measure by Bonacich (1987) are ecological. Experimental results are consistent in every case with the notion that power is ecological, but two experiments violate the principle of regularity. Unfortunately, neither experiment is definitive in this respect, so final judgement on regularity awaits a result in which a statistically significant difference is observed between two nodes that, according to regularity, must be equally powerful.

Three clear areas for further research are evident. First, we can generalize the approach to handle variable numbers of exchanges per round. Second, we can develop a parallel approach for handling ordinal rather than nominal power relations. Third, we can systematically evaluate other theories of power to see whether they conform to our propositions.

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