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Exact colorations of graphs and digraphs

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Abstract

A coloration is an exact regular coloration if whenever two vertices are colored the same they have identically colored neighborhoods. For example, if one of the two vertices that are colored the same is connected to three yellow vertices, two white and red, then the other vertex is as well. Exact regular colorations have been discussed informally in the social network literature. However they have been part of the mathematical literature for some time, though in a different format. We explore this concept in terms of social networks and illustrate some important results taken from the mathematical literature. In addition we show how the concept can be extended to ecological and perfect colorations, and discuss how the CATREGE algorithm can be extended to find the maximal exact regular coloration of a graph.

1. Introduction

The use of regular equivalences to formally capture the intuitive notions of position and social role is well established in the social networks literature. Regular equivalence provides an important theoretical underpinning for a variety of practical tools for data analysis. The richness of the idea, first published by White and Reitz (1983), became clear when it was realized that the concept defines a lattice of equivalences (Borgatti and Everett, 1989) which includes both structural equivalence (Lorrain and White, 1971) and automorphic equivalence (Winship and Mandel, 1983; Everett, 1985) as members. Everett and Borgatti (1991) reformulated the concept in terms of graph coloration. A review and synthesis of the mathematical properties of regular coloration can be found in Everett and Borgatti (1994a).

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The major criticism of structural equivalence as a model for social role is that for two individuals to be equivalent they must be tied to exactly the same others. Maximal regular coloration overcomes this problem but has practical utility only when applied to digraphs; the maximal regular coloration of a connected undirected graph is trivial. Automorphic equivalence is a method which is applicable to graphs and digraphs but is computationally difficult. While some algorithms have been proposed (e.g. in UCINET, Borgatti et al., 1992 or Sparrow, 1993) these algorithms, except in very small networks, cannot guarantee to find the orbits when they exist, nor do they provide easily interpretable cost functions for measuring how close to an orbit partition a given partition is. The latter consideration is important in the design of combinatorial optimization algorithms.

In this paper we explore a regular coloration which is not necessarily trivial for graphs can be found by an iterative method and can be formulated as a problem in combinatorial optimization. The concept is less strict than automorphic equivalence and is therefore more common. We also consider some additional related colorations that are not regular.

Before giving a formal definition it is necessary to introduce some notation. Let $G(V, E)$ be a finite graph with vertex set V and edge set E. The edges can be directed and both self loops and multiple edges are allowed. If $v \in V$ then the in degree of v, denoted by $\rho_i(v)$, is the number of edges which terminate at v. The out-degree, $\rho_o(v)$, is the number of edges which initiate from v.

A coloration C is an assignment of colors to the vertices of G. If S is a subset of V then the spectrum of S, denoted by $C(S)$, is the set of colors assigned to the vertices in S. Additionally, if k is a color, then the k in-degree of v, denoted by ${}^k\rho_i(v)$ is the number of edges which initiate from a vertex colored k and terminate with v. The k out-degree, ${}^k\rho_o(v)$, is the number of edges which initiate from v and terminate at a vertex colored k. In the graph in Fig. 1 we see that $\rho_o(2) = 3$, $\rho_i(4) = 6$, ${}^y\rho_o(5) = 2$ and ${}^R\rho_i(4) = 5$.

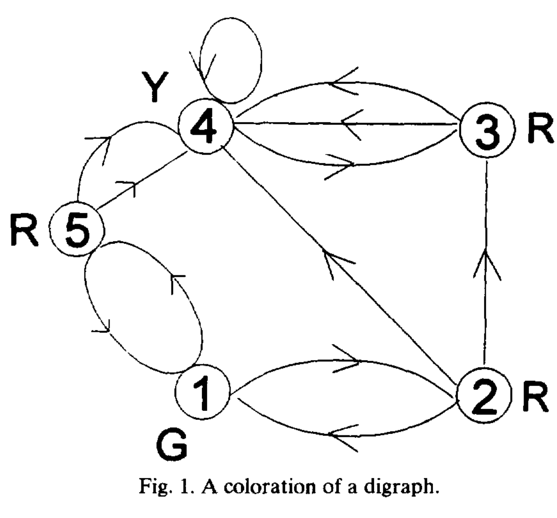

Fig. 1. A coloration of a digraph.

A coloration C of a graph $G(V, E)$ is an exact regular coloration if and only if for all $u, v \in V$ and every $k \in C(V)$

$$C(u) = C(v) \Rightarrow {}^k\rho_i(u) = {}^k\rho_i(v) \text{ and } {}^k\rho_o(u) = {}^k\rho_o(v)$$

If G is not directed then clearly these last two conditions are equal and reduce to

$${}^k\rho(u) = {}^k\rho(v)$$

Exact regular colorations in the context of social networks were first discussed by Borgatti and Everett (1992) but were not formally defined until later in Everett and Borgatti (1994b) where they were called exact colorations. However, the concept has been part of the graph theory literature for nearly 30 years where it is known as the divisor concept (Sachs, 1966).

Two actors are exactly regularly equivalent if they have neighborhoods with identical colors. Clearly exactly regularly equivalent actors are regularly equivalent but the converse is false. As a practical example, consider the biological parent relationship. Structural coloration would class together only children of the same mother (i.e. brothers and sisters). Each mother would be placed in her own class. Maximal regular coloration would put together all mothers in one class and all children in the other. Like automorphic coloration, exact regular coloration would put together all mothers with the same number of children and all children with the same number of brothers and sisters.

Fig. 1 is not an exact regular coloration. For example $C(4) = C(3)$ but ${}^R\rho_i(4) = 5$ whereas ${}^R\rho_i(3) = 1$. The graph shown in Fig. 2 is an exact regular coloration. We see that each vertex colored R is connected to two vertices colored B and one vertex colored R and each B vertex is connected to one vertex colored B and two vertices colored R.

Any coloration based upon structural equivalence will result in an exact regular coloration. Also the orbits of any subgroup of the automorphism group of a graph will induce an exact regular coloration. The concept is, however, more general than automorphic equivalence. This is shown by the graph in Fig. 2. As already

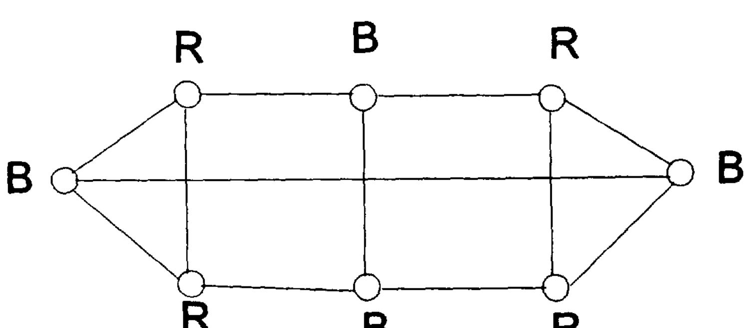

Fig. 2. An exact regular coloration.

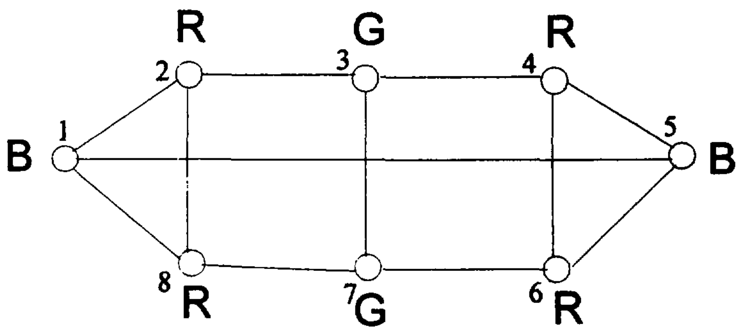

Fig. 3. An alternative exact regular coloration of the graph in Fig. 2.

stated, this coloration is an exact regular coloration, but since two vertices colored B are in cycles of length 3 and the two other vertices colored B are not, the coloration cannot have been derived from the orbit structure of any subgroup.

2. Properties

A graph or digraph may have more than one exact regular coloration. For example, the graph in Fig. 3 is a different exact regular coloration from the one in Fig. 2. In addition, coloring each vertex the same would be exact regular (since the graph is regular) and coloring each vertex differently is vacuously exactly regular. It is well known that the set of all regular colorations form a lattice under the refinement relation (Borgatti and Everett, 1989). This property carries over to exact regular colorations.

Theorem 1 (Everett and Borgatti, 1994). The class of all exact regular colorations form a lattice under the refinement relation.

The minimum element of this lattice is the trivial coloration in which every vertex is colored differently. We shall call the maximal element the *maximal exact regular coloration*. The maximal exact regular coloration of a regular graph will simply be the coloration which colors all vertices the same. However, for any non-regular graph this will not be the case. The graph in Fig. 4 has a non-trivial maximal exact regular coloration which is not derived from the orbit structure nor is it simply a coloring of the vertices by degree. This is easily seen since all red and green vertices have the same degree but two of the red vertices are on cycles of length three and two of them are not. As already stated, exact regular coloration can be viewed as a relaxation of automorphic equivalence. It shares with automorphic equivalence the important property that the equivalance classes for an undirected graph are not necessarily trivial. However, this in itself would not make it preferable to automorphic equivalence. Its strength lies in the fact that, unlike automorphic equivalence, we can adapt standard algorithms to find the maximal

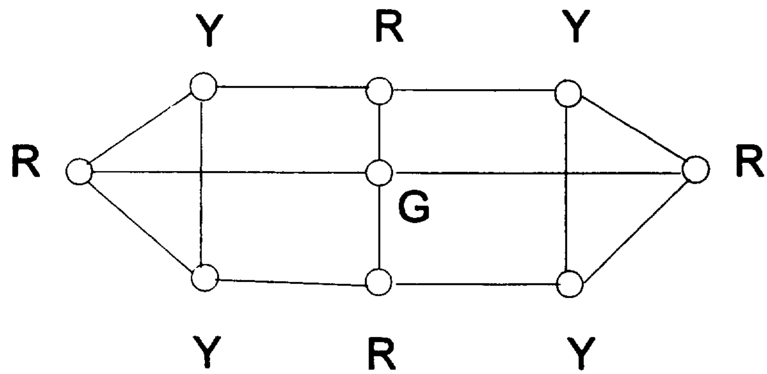

Fig. 4. A maximal exact coloration not derived from the orbits.

element of the lattice and develop cost functions which can be used in combinatorial optimization algorithms.

We can capture the essential feature of exact regular equivalence in an enhanced image graph. Let $G(V, E)$ be a graph with an exact regular coloration C. The *enhanced image graph* $G'(C(V), E')$ has the spectrum of V as its vertices. Two vertices A and B in the image graph have an edge from A to B of weight w if a vertex colored A is connected to w vertices colored B in G. Note that an undirected graph gives rise to a weighted digraph as an enhanced image graph. Fig. 5 gives the enhanced image graph for the regular coloration given in Fig. 3. Note also that the colors assigned to the vertices are no longer a coloration but are simply labels. We can now color the image vertices and this coloration of the image could also be regular. Note that the weights on the edges give their multiplicity and this must be taken into account when deciding whether a coloration is exact regular or not. Fig. 6 gives a regular coloration of the enhanced image graph in Fig. 5. Suppose that C is a coloration of $G(V, E)$ and C' is a coloration of the enhanced image $G'(C(V), E')$. Then for every vertex $v \in V$ the *composite coloration* $C' \circ C$ assigns the color $C'(C(v))$. The composition of the colorations in Figs. 3 and 6 give the coloration shown in Fig. 2. We note that this composition

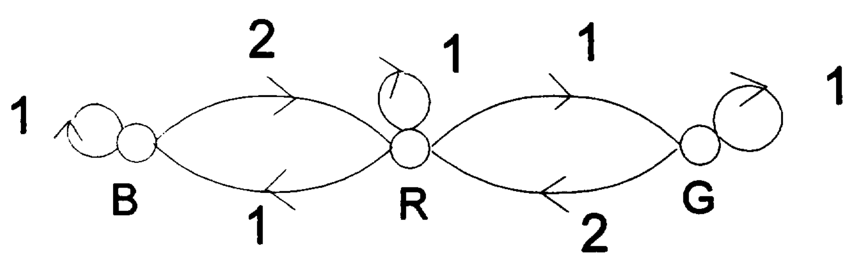

Fig. 5. The enhanced image graph of the coloration in Fig. 3.

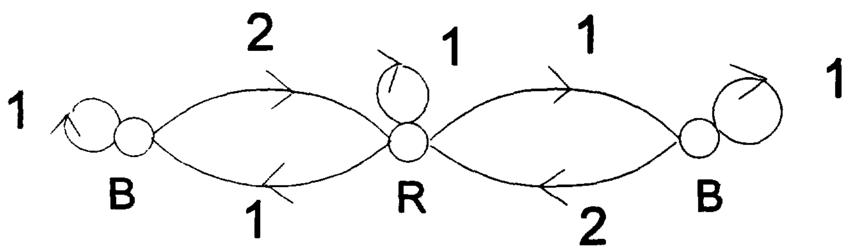

Fig. 6. An exact coloration of the enhanced image graph in Fig. 5.

results in another exact regular coloration of the graph in Fig. 3. This property holds in general and was noted by the developers of the divisor concept.

Theorem 2 (Cvetorick et al., 1987). If C is an exact regular coloration of a graph G and C' is an exact regular coloration of the enhanced image graph G' then $C'^{\circ}C$ is an exact regular coloration of G.

The interest shown by mathematicians in exact regular coloration derives from an important property of the enhanced image graph. The eigenvalues and characteristic polynomial of a graph are defined to be the eigenvalues and characteristic polynomial of its adjacency matrix.

Theorem 3 (Sachs, 1966). The characteristic polynomial of the enhanced image graph of an exact regular coloration divides the characteristic polynomial of the original graph.

It is for this reason that exact regular coloration is known as the divisor concept in the graph theory literature. In fact, a stronger version of this theorem was proved. A coloration C is an exact out-regular coloration if and only if for all u, $v \in V$ and every $k \in C(V)$

$$C(u) = C(v) \Rightarrow {}^k\rho_o(u) = {}^k\rho_o(v)$$

We can replace exact regular coloration by exact out-regular coloration and the theorem remains true.

We can illustrate the above theorem by examining the eigenvalues of the graph in Fig. 3, the graph in Fig. 5 and the graph in Fig. 7. The graph in Fig. 7 is the

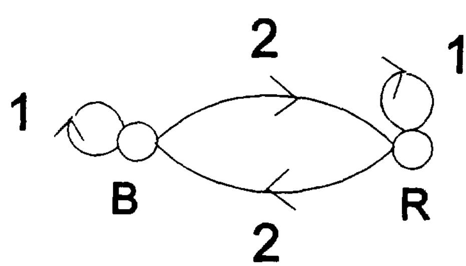

Fig. 7. The enhanced image of the coloration in Fig. 6.

	1	5	2	4	6	8	3	7
1	0	1	1	0	0	1	0	0
5	1	0	0	1	1	0	0	0
2	1	0	0	0	0	1	1	0
4	0	1	0	0	1	0	1	0
6	0	1	0	1	0	0	0	1
8	1	0	1	0	0	0	0	1
3	0	0	1	1	0	0	0	1
7	0	0	0	0	1	1	1	0

Fig. 8. Blockmodel corresponding to the exact regular correlation in Fig. 3.

enhanced image of the exact regular coloration (see Fig. 6) of the graph in Fig. 5, which is itself the image graph of the exact regular coloration shown in Fig. 3. The eigenvalues of the graphs in Figs. 3, 5, and 7 are, respectively, $\{3, \pm\sqrt{3}, \pm 1, -1 \pm \sqrt{2}\}$, $\{\pm 1, 3\}$, and $\{-1, 3\}$. As predicted, each set of eigenvalues is a subset of the preceding set.

Given a coloration of the vertices of a graph, it is common in network analysis to permute the rows and columns of the adjacency matrix of the graph so as to group together vertices that are colored the same. The partitioned and permuted matrix is often called a blockmodel. A blockmodel induces a partition of the cells of the adjacency matrix into *matrix blocks*, as shown in Fig. 8. We can now characterize exact regular coloration in terms of the induced blocks of the adjacency matrix. The proof of the following theorem is straightforward.

Theorem 4. A coloration of a graph is an exact regular coloration if and only if each block of the induced partition has every row sum the same and every column sum the same.

Note that within any block the row sums may be different from the column sums, as is the case in Fig. 8. It is the value of the row sums which form the entries of the adjacency matrix of the enhanced image graph.

This formulation allows us to define a cost function which measures to what extent a given coloration is an exact regular coloration. This function merely sums the minimum number of changes required to make each block fulfil the condition. This can be submitted to a combinatorial optimization routine. This approach has been used successfully for small networks to find certain regular colorations (Batagelj et al., 1992).

3. Exact regular coloration algorithms

Combinatorial optimization methods, whilst useful for small networks have some important shortcomings. Firstly, the user must specify the number of blocks (i.e. the number of colors in the coloration). Secondly there is no guarantee that the algorithm will find the global minimum of the cost function. Finally, the method is not efficient on large networks. In this section we show how the algorithm CATREGE (Borgatti and Everett, 1993) can be adapted to find the maximal exact regular coloration. This new algorithm, called EXCATRE, is of the order of n^3. It can find the maximal exact regular coloration of, say a 250 vertex graph in one or two seconds on a PC. A combinatorial optimization program could take hours, even days, to complete this task.

The EXCATRE algorithm creates a series of increasingly refined colorations, beginning with the one in which all nodes are colored the same and ending with the maximal exact regular coloration. The color of a node v in the tth iteration is written $C^t(v)$.

1. Set iteration counter to zero ($t \leftarrow 0$) and assign all nodes the same color for the initial coloration ($C^t(j) \leftarrow 1$ for all j).
2. Assign each node a unique color for the next coloration ($C^{1+1}(f) \leftarrow j$ for all j) and set FLAG ← FALSE.
3. For every pair of nodes u, v colored the same in coloration t, compare their k-in-degree and k-out-degree for all k. If they are exactly the same, then reassign $C^{1+1}(u) \leftarrow C^{t+1}(v)$, else set FLAG ← TRUE.
4. If FLAG ← TRUE then increment counter t and goto step 2.

A benefit of the algorithm is the sequence of intermediate colorations, which may be seen as approximate or nearly exact regular colorations. This is useful for analyzing graphs in which the maximal exact regular colorations is the identity partition in which all nodes are assigned a unique color. For example, if we apply the algorithm to John Padgett's data on marriages between medieval Florentine families (reported by Breiger and Pattison, 1986), coloration C^1 divides the families into six classes, including two singletons (see Fig. 9). The next coloration, C^2, places all families in their own singleton classes except Ridolfi and Tornabuoni, who remain assigned the same color. The final coloration yields the (trivial) maximal exact regular coloration. As with CATREGE, the number of iterations needed to separate two vertices can be taken as a simple measure of the extent of exact regular equivalence between the vertices.

This algorithm can be viewed as a special case of a general coloring algorithm. The general coloring algorithm can be used for any coloration of the form

$$C(u) = C(v) \Rightarrow \text{ condition } u,v \in V$$

This class includes regular, exact regular and automorphic equivalence, as well as non-regular colorations such as in and out regular and in and out exact regular. For definitions and a review of the mathematical properties of these equivalence see Everett and Borgatti (1994a). The general coloring algorithm proceeds as follows:

0. Initially assign all vertices the same color.
1. For each distinct color κ presently in use:
   (a) choose any vertex μ that is colored κ,
   (b) define a new color η not previously used,

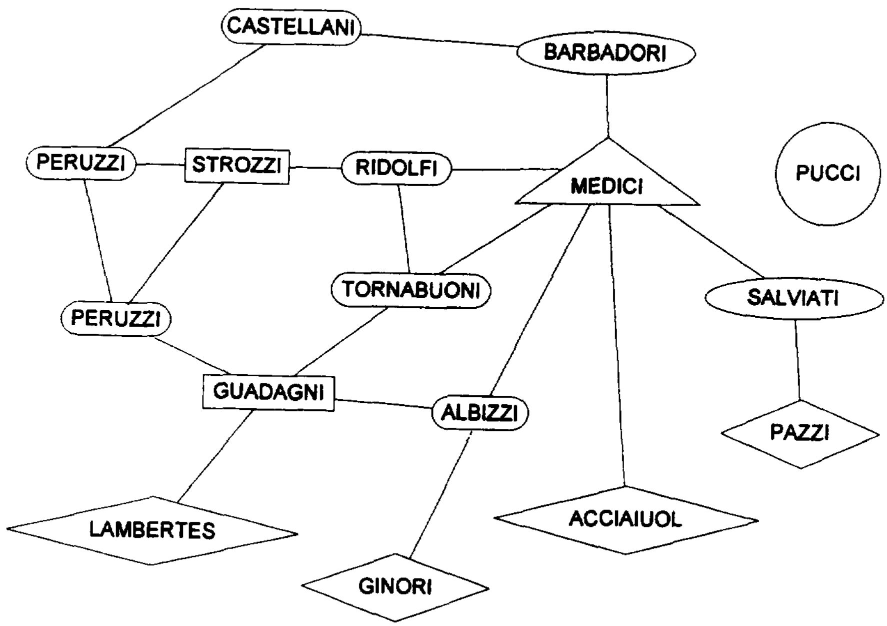

Fig. 9. Equivalence classes after the first iteration of the ExCatRe algorithm, applied to Padgett's marriage data.

(c) test the equivalence condition on every other vertex colored κ,
(d) for all vertices v that fail the test, let $C(v) = \eta$.
2. Repeat Step 1 until no further recolorings occur.

The above algorithm will terminate after at most n iterations (where n is the number of vertices) of Step 1. This step can be executed in $O(n^2)$ time for the exact regular coloration condition (this is the same as EXCATRE) so that the algorithm has $O(n^3)$. Convergence will be to the maximum equivalence i.e. the equivalence which requires the fewest number of colors. It should be noted that whilst this algorithm can be used in principle to find the orbits it would not be practicable. To check the condition we would need to know the orbits already, hence it would be n times more inefficient than a straight orbit finder. (All such algorithms are exponential.)

We illustrate the algorithm on the graph in Fig. 4 (we remove the colors assigned to the vertices). At Step 0, we assign the color RED to every vertex. At Step 1, we arbitrarily choose vertex 3 as our reference point, and compare the neighborhoods of all vertices to the neighborhood of vertex 3. Since the vertex is connected to three RED vertices the only vertex to fail the condition is vertex 5 which we re-color GREEN. We now move to Step 2 which sends us back to Step 1.

Starting with the RED vertices, our first fixed vertex is again 3 and it is now adjacent to two Red (1 and 2) and one Green (5) vertex. We note that the Red vertices 4, 6 and 7 satisfy this condition but 1, 2, 8 and 9 do not. We therefore re-color these vertices Yellow and fix vertex 1 (say). We now consider the next fixed vertex, 5, which is colored Green. As this is the only Green vertex we do not have to perform the task and therefore all fixed vertices have now been considered. We again repeat the procedure in Step 1. This iteration does not involve any re-coloring so the algorithm terminates with the exact regular coloration given in Fig. 4.

The efficiency of this algorithm can be improved by implementing more sophisticated re-coloration and an improved starting coloration. (We could for example color each vertex according to its degree.) One shortcoming of the algorithm is that we would not generally find the intermediate colorations to be interpretable, so that no measure of the extent of equivalence between vertices is possible.

4. Exact perfect colorations

Borgatti and Everett (1992) introduce the concept of ecological coloration to try and capture the idea that an actor's social role is determined by their environment. This concept is developed further in Borgatti and Everett (1994). In the latter paper they give the following definitions.

A coloration C is *ecological* if $\forall\ a,b \in V$ whenever $C(N_i(u)) = C(N_i(v))$ and $C(N_0(u)) = C(N_0(v))$ then $C(u) = C(v)$. Ecological colorations are precisely the converse of regular colorations. A coloration which is both regular and ecological is called *perfect*.

An *exact perfect coloration* is an ecological coloration which is also an exact regular coloration. The coloration given in Fig. 4 is an exact perfect coloration. Structural equivalence is also an exact perfect coloration. The exact regular coloration given in Fig. 2 is not an exact perfect coloration. We can again adapt theorems given by Borgatti and Everett on perfect colorations so that they apply to exact perfect colorations.

Theorem 5. An exact regular coloration is an exact perfect coloration if and only if the image graph contains no structurally equivalent vertices.

Theorem 6. The set of all exact perfect colorations of a graph, ordered by the refinement relation, forms a lattice.

5. Exact ecological colorations

We can extend the concept to exact ecological colorations. An exact ecological coloration is one in which we insist that vertices with identically colored neighborhoods must be colored the same. Formally a coloration C of a graph $G(V, E)$ is an exact ecological coloration if and only if for all $u, v \in V$ and every $k \in C(V)$ if

$$^{k}\rho_i(u) = {}^{k}\rho_i(v)$$

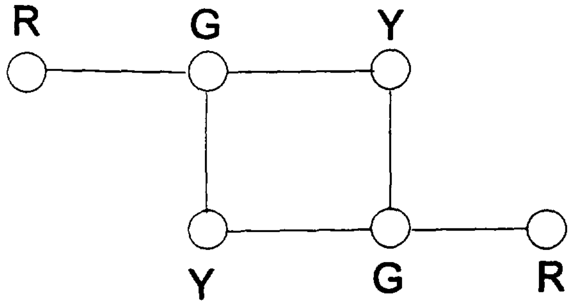

Fig. 10. An exact ecological coloration which is not ecological.

and

$$^{k}\rho_{o}(u) = {}^{k}\rho_{o}(v)$$

then

$$C(u) = C(v)$$

This condition is a weakening of the ecological colorations. All ecological colorations are exact ecological but the converse is not true. The coloration given in Fig. 10 is an exact ecological coloration which is not ecological. This can be seen by noting that the neighborhoods of the Red vertices are Green and the colors of the Yellow vertices' neighborhoods are also Green (but 2 Greens instead of 1).

Note that an exact perfect coloration, defined in the previous section, is *not* a regular coloration which is also an exact ecological coloration.

As with the other colorations we have introduced, the set of exact ecological colorations forms a lattice.

Theorem 7. The set of all exact ecological colorations ordered by the refinement relation, form a lattice.

Proof. Similar to the lattice proof for ecological colorations in Borgatti and Everett, 1994 (Theorem 5). □

The model of exact ecological coloration captures the property that it is not just the connections to individuals with certain properties that mould an individual's role, but the number of such individuals. This view is consistent with the network models of contagion (Burt, 1992) in which importance is placed on the number of actors in the neighborhood with certain attributes. For example, we might expect a social environment which includes 25 criminals to exert a different social pressure than an environment which includes only one criminal, therefore we would not necessarily expect the same outcomes for two individuals with such different environments (unlike the model of ecological coloration).

Exact ecological colorations can be used to extend the work of Borgatti and Everett (1992) on experimental exchange networks (Cook et al., 1983; Markovsky

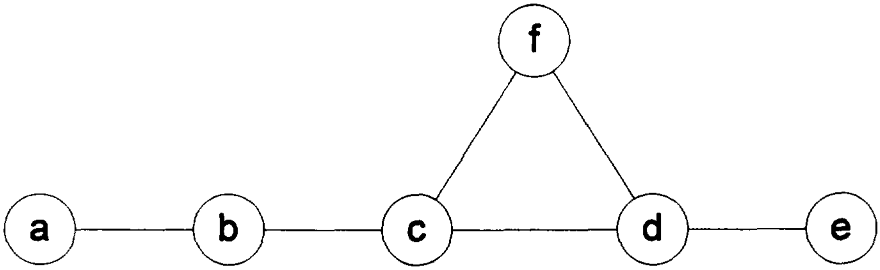

Fig. 11. An exchange network.

et al., 1988). In these experiments, actors embedded in different opportunity networks bargain with each other for points. If we regard actors who trade at parity to be equivalent, we induce a coloration on the nodes of the opportunity network. Borgatti and Everett suggested on theoretical grounds that such colorations would be ecological because the bargaining power of an actor is a function of the power of the actors in their neighborhood – an actor surrounded by only weak actors will have power, whereas an actor surrounded by only strong actors will not.

However, preliminary results from a more recent study by Skvoretz and Willer [1] on the graph in Fig. 11 cast some doubt on this theory. They found, pending statistical analysis, that only nodes c and d clearly trade at parity. The coloration induced $(\{\{a\}\{b\}\{c, d\}\{e\}\{f\}\})$ is not ecological, because both f and e are surrounded by the same color neighborhoods, yet they are assigned different colors themselves. However, the coloration is an exact ecological coloration, since f is connected to two elements while e is connected to just one. The suggestion is that ecological coloration is too strong a model for exchange experiments because it considers only the types of colors in a node's neighborhood, and not the frequency. Hence it contradicts the common sense assumption that being connected to five weak nodes is no more advantageous than being connected to one weak node. Exact ecological coloration corrects this deficiency. [2]

6. Conclusion

Over the last 25 years of social network research, two major families of equivalence concepts have been advanced; the regular and ecological colorations. The fundamental theme of this work is that actors are classified in terms of their connections to types of actors. However, little attention has been paid to the

[1] This work is unpublished as yet. Preliminary results were reported by Borgatti (1994) with permission. The present authors are grateful to Skvoretz and Willer for allowing us to use these results in this paper as well.

[2] The authors are indebted to James Boster for criticizing ecological coloration in this way.

number of connections to each type of actor. In this paper, we have introduced that consideration for both regular and ecological colorations. In the case of regular colorations, this means narrowing our focus down to a subset of regular colorations. In the case of ecological colorations, this means widening our focus up to a superset of colorations that include ecological colorations as members. Both exact regular colorations and exact ecological colorations provide stronger links to existing mathematical theory, and may provide better models for empirical network phenomena.

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