Cohesion

Dyadic and Whole Network

Dyadic vs Whole Network

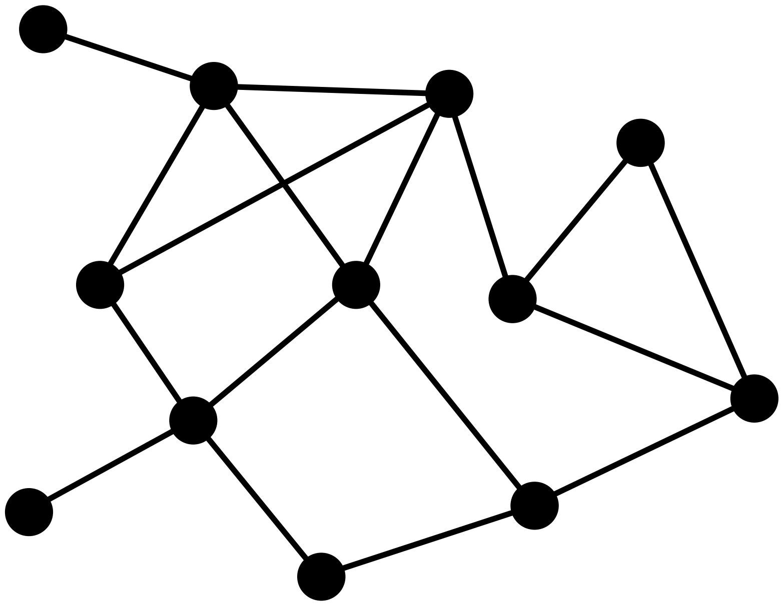
- Dyadic cohesion refers to pairwise social closeness
- Whole network measures can be
  - Averages of dyadic cohesion
  - Measures not easily reducible to dyadic measures

Measures of Group Cohesion

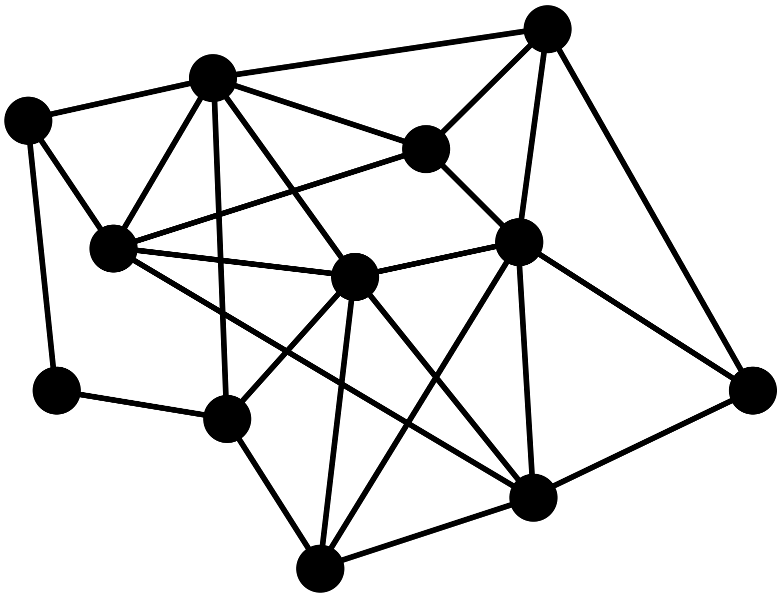
- Density & Average degree
- Average Distance and Diameter
- Number of components
- Fragmentation
- Distance-weighted Fragmentation
- Cliques per node
- Connectivity
- Centralization
- Core/Peripheriness
- Transitivity (clustering coefficient)

Density

- Number of ties, expressed as percentage of the number of ordered/unordered pairs

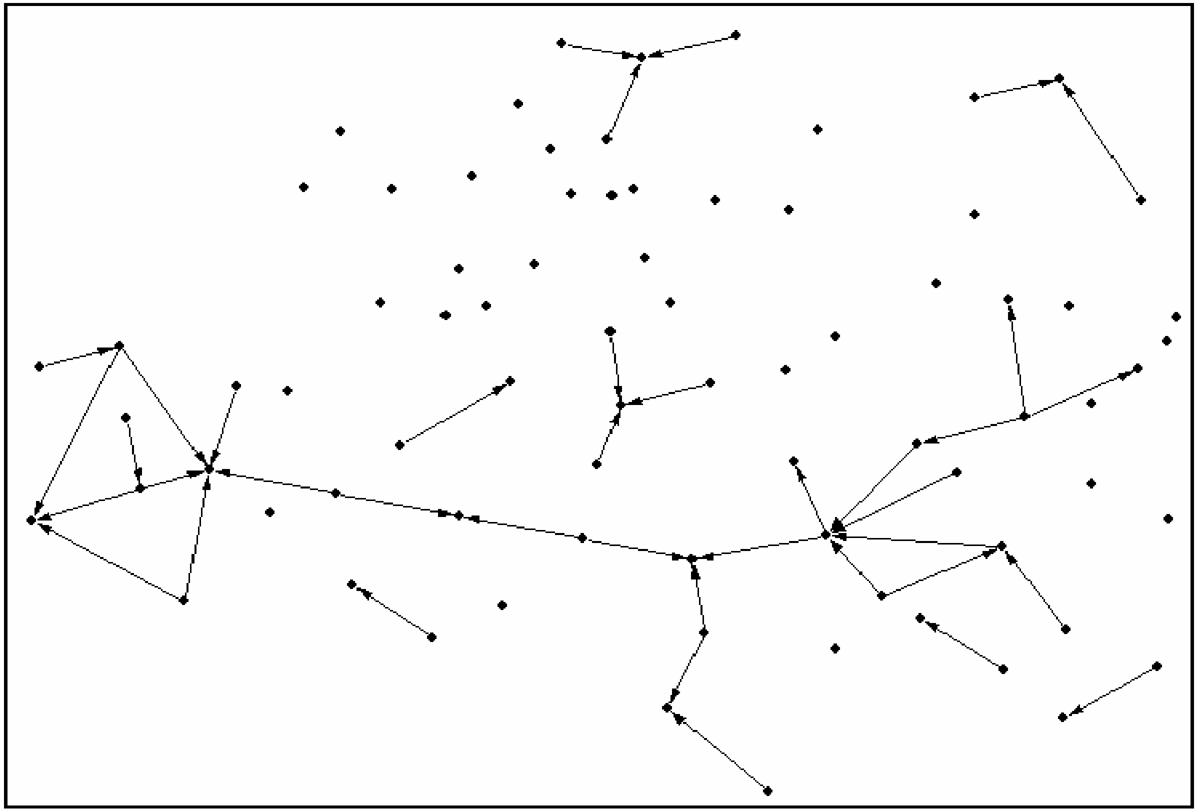


Low Density (25%)
Avg. Dist. = 2.27



High Density (39%)
Avg. Dist. = 1.76

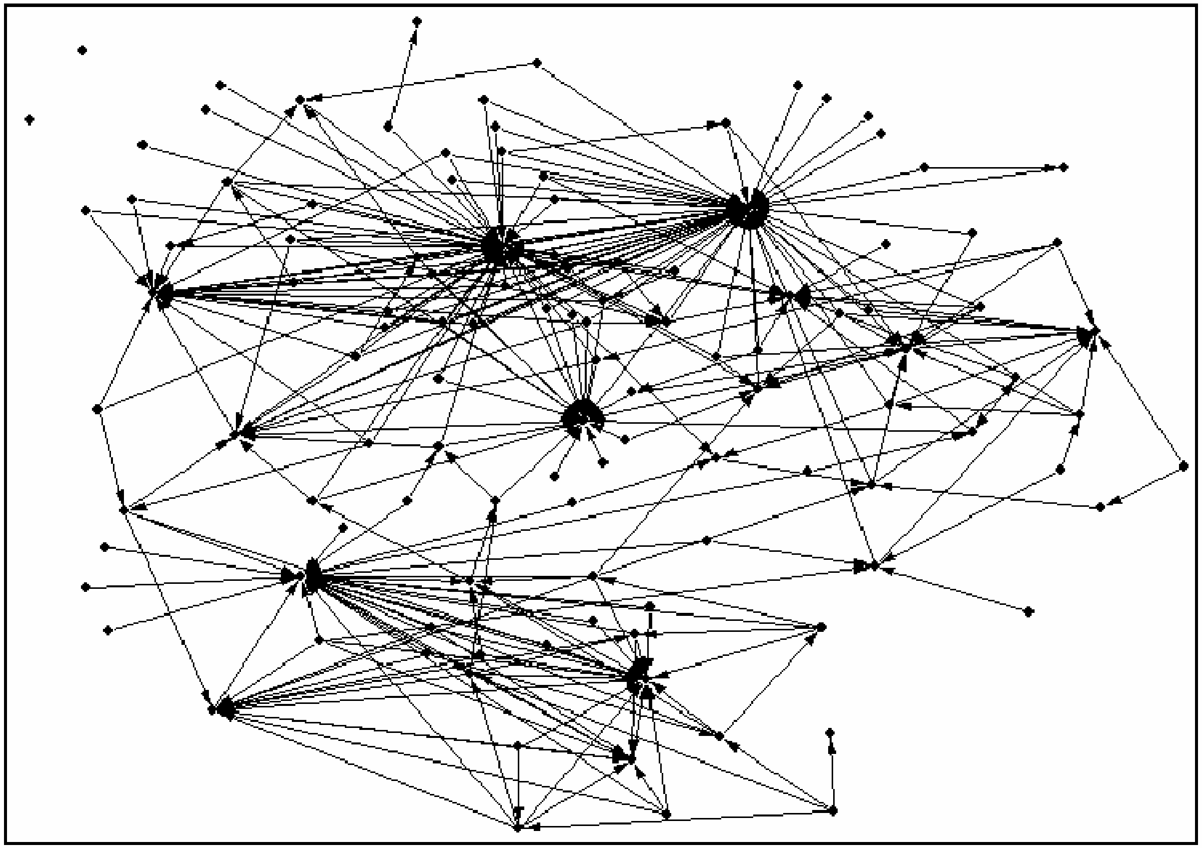
Help With the Rice Harvest



Village 1

Data from Entwistle et al

Help With the Rice Harvest



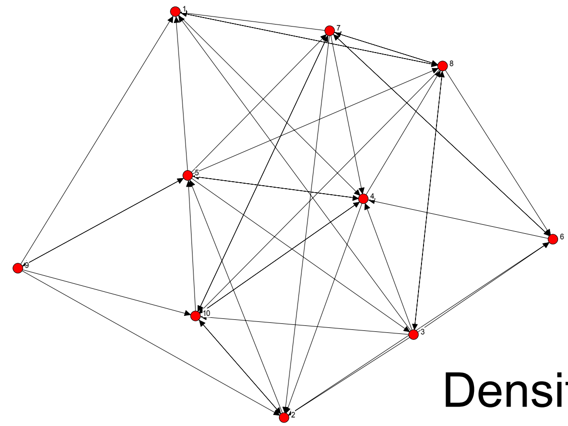
Village 2

Which village is more likely to survive?

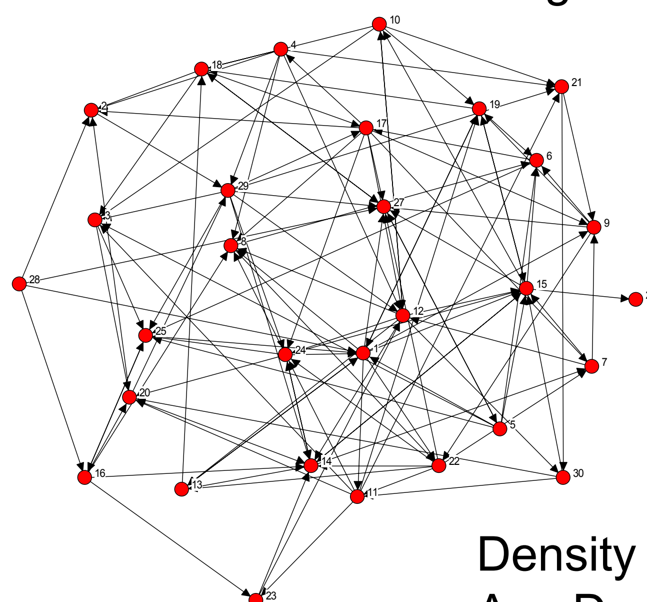
Data from Entwistle et al

Average Degree

- Average number of links per person
- Is same as density*(n-1), where n is size of network
  - Density is just normalized avg degree – divide by max possible
- Often more intuitive than density



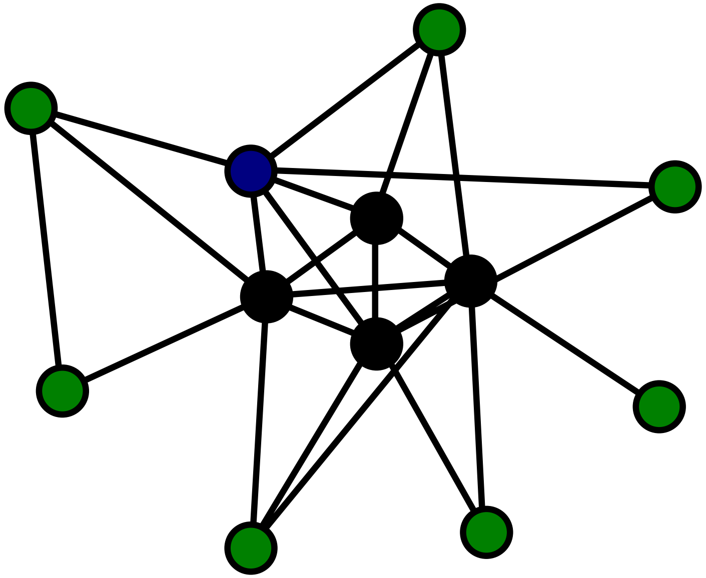
Density 0.47
Avg Deg 4



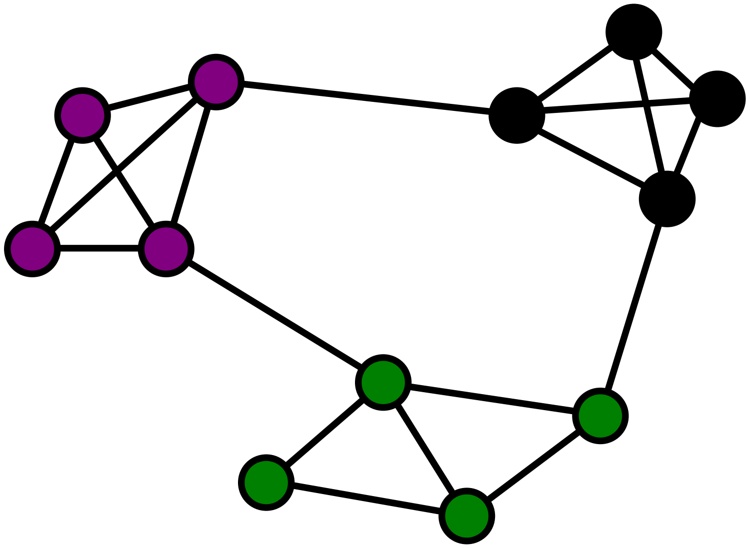
Density 0.14
Avg Deg 4

Average Distance

- Average geodesic distance between all pairs of nodes



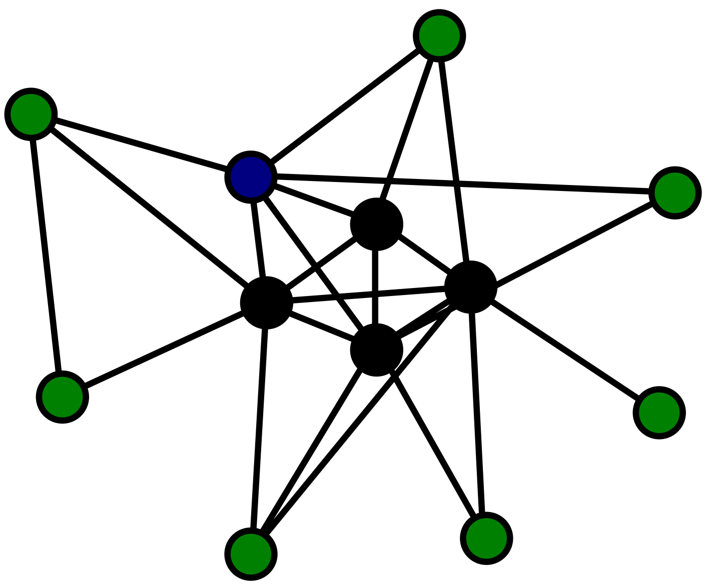
avg. dist. = 1.9



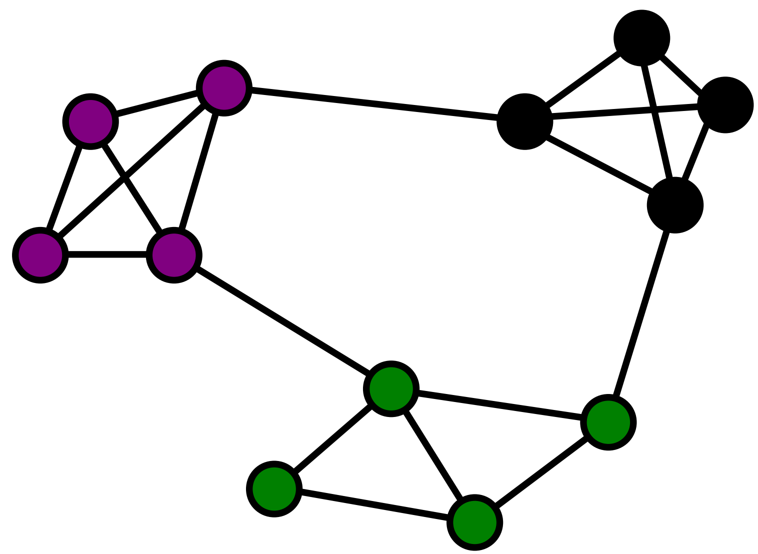
avg. dist. = 2.4

Diameter

- Maximum distance



Diameter = 3



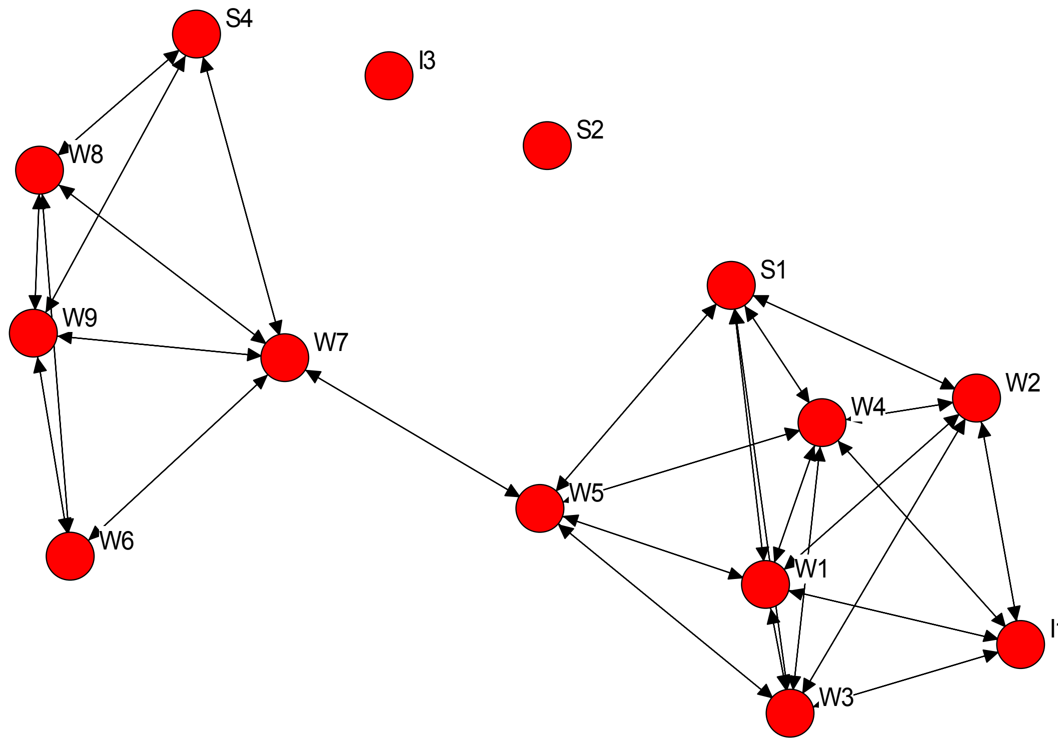
Diameter = 3

Fragmentation Measures

- Component ratio
- F measure of fragmentation
- Distance-weighted fragmentation ${}^{D}F$

Component Ratio

- No. of components divided by number of nodes



Component ratio = 3/14 = 0.21

F Measure of Fragmentation

- Proportion of pairs of nodes that are unreachable from each other

$$F = 1 - \frac{2\sum_{i>j} r_{ij}}{n(n-1)}$$

r_{ij} = 1 if node i can reach node j by a path of any length
r_{ij} = 0 otherwise

- If all nodes reachable from all others (i.e., one component), then F = 0
- If graph is all isolates, then F = 1

Computation Formula for F Measure

- No ties across components, and all reachable within components, hence can express in terms of size of components

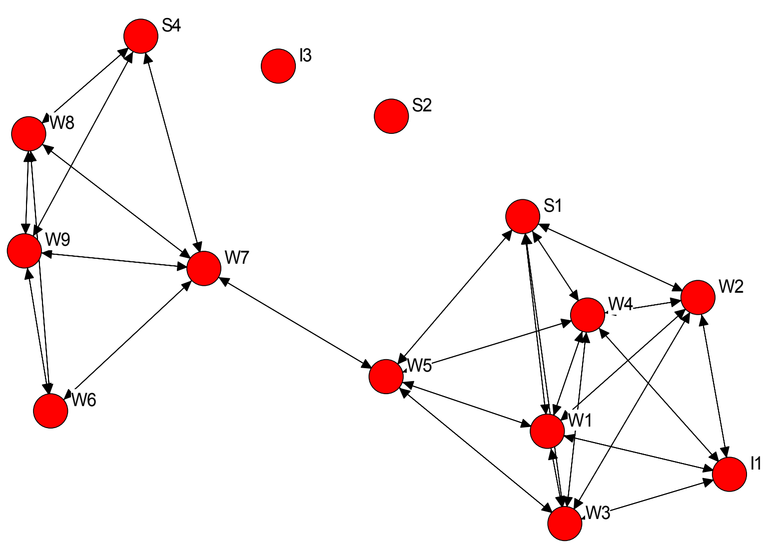
$$F = 1 - \frac{\sum_k s_k (s_k - 1)}{n(n-1)}$$

Sk = size of kth component

Computational Example

Games Data

Comp	Size	Sk(Sk-1)
1	1	0
2	1	0
3	12	132
	14	132
		0.2747 = 14/(132*131) = F



Heterogeneity/Concentration

- Sum of squared proportion of nodes falling in each component, where s_k gives size of kth component:

$$H = 1 - \sum_k \left(\frac{s_k}{n} \right)^2$$

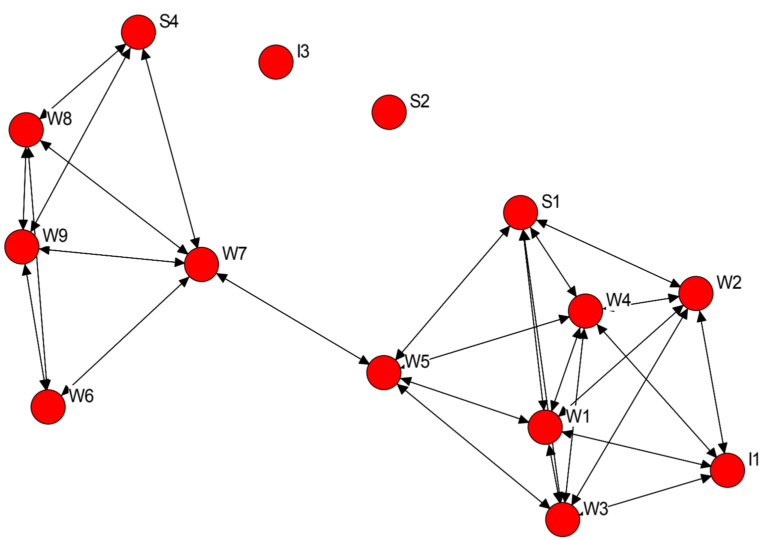
- Maximum value is 1-1/n
- Can be normalized by dividing by 1-1/n. If we do, we obtain the F measure

$$F = 1 - \frac{\sum_k s_k (s_k - 1)}{n(n-1)}$$

Heterogeneity Example

Games Data

Comp	Size	Prop	Prop^2
1	1	0.0714	0.0051
2	1	0.0714	0.0051
3	12	0.8571	0.7347
	14	1.0000	0.7449



Heterogeneity = 0.255

Distance-Weighted Fragmentation

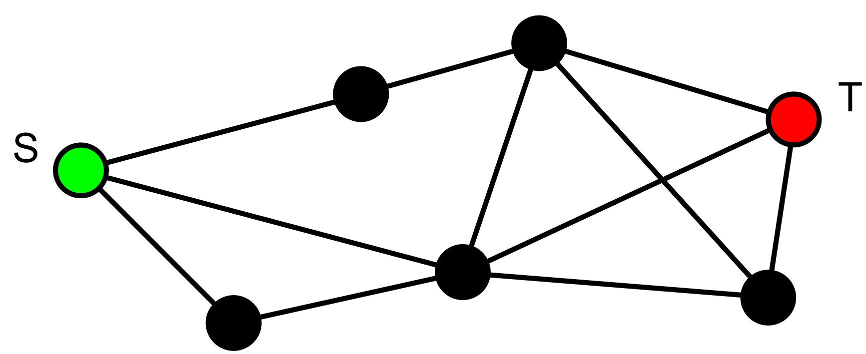
- Use average of the reciprocal of distance
  - letting $1/\infty = 0$

$$^{D}F = 1 - \frac{2\sum_{i>j} \frac{1}{d_{ij}}}{n(n-1)}$$

- Bounds
  - lower bound of 0 when every pair is adjacent to every other (entire network is a clique)
  - upper bound of 1 when graph is all isolates

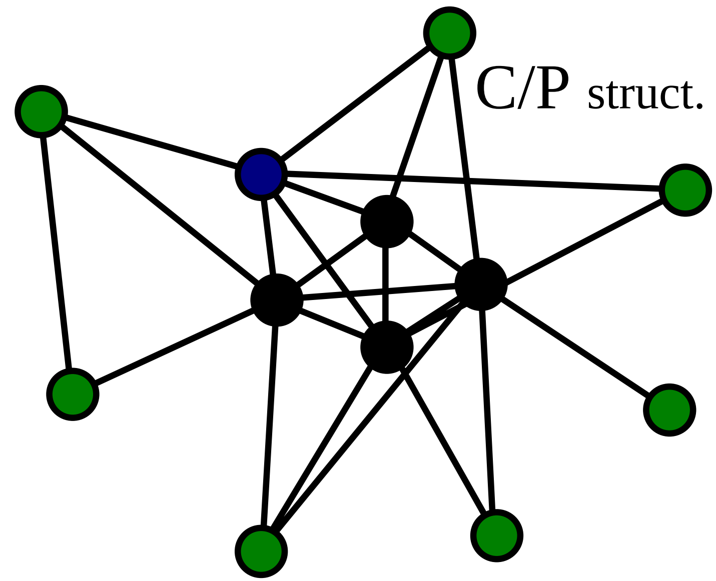
Connectivity

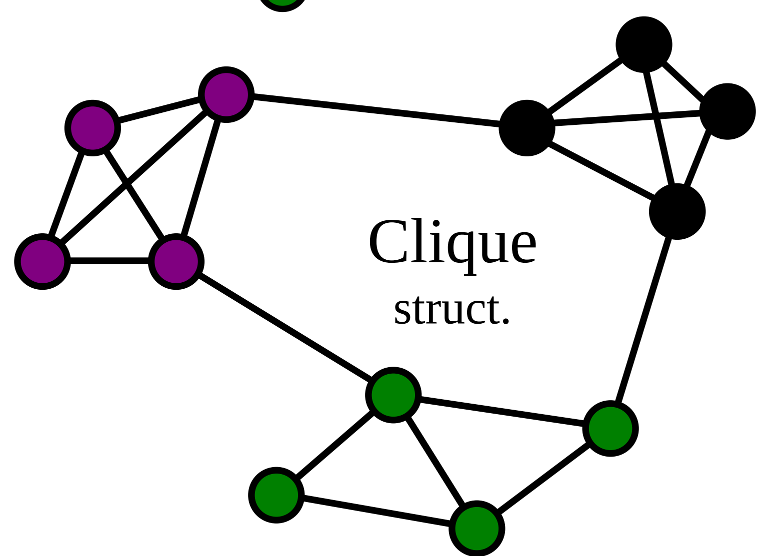
- Line connectivity λ is the minimum number of lines that must be removed to discon-nect network
- Node connectivity κ is minimum number of nodes that must be removed to discon-nect network



Core/Periphery Structures

- Does the network consist of a single group (a core) together with hangers-on (a periphery), or
- are there multiple sub-groups, each with their own peripheries?

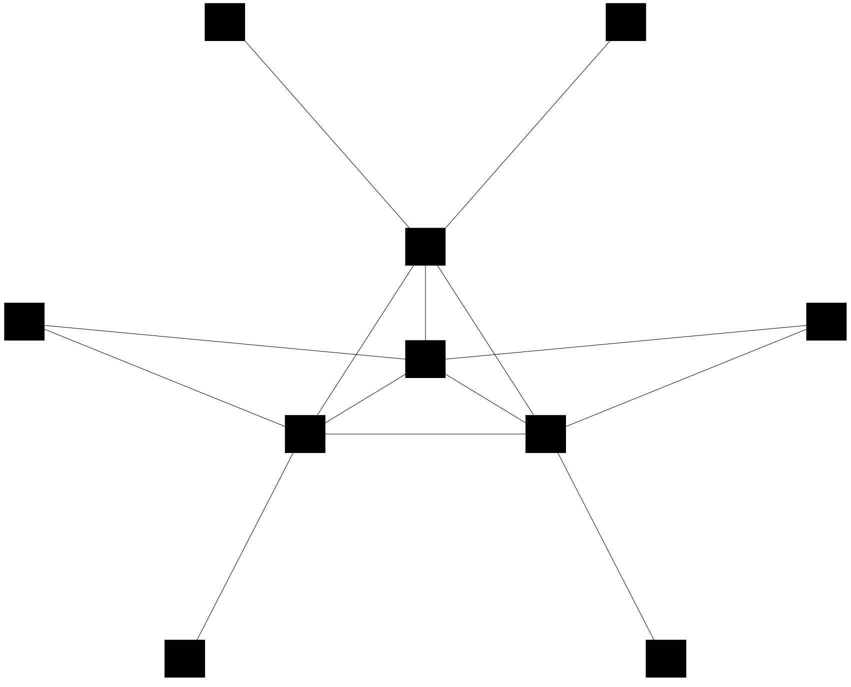




Kinds of CP/Models

- Partitions vs. subgraphs
  - just as in cohesive subgroups
- Discrete vs. continuous
  - classes, or
  - coreness

A Core/Periphery Structure



Blocked/Permuted Adjacency Matrix

	CORE				PERIPHERY					
	-	1	1	1	1	0	0	1	0	0
	1	-	1	1	0	1	1	0	0	0
CORE	1	1	-	1	0	0	0	1	1	0
	1	1	1	-	1	0	0	0	0	1
	1	0	0	1	-	0	0	0	0	0
	0	1	0	0	0	-	0	0	0	0
PERIPHERY	0	1	0	0	0	0	-	0	0	0
	1	0	1	0	0	0	0	-	0	0
	0	0	1	0	0	0	0	0	-	0
	0	0	0	1	0	0	0	0	0	-

- Core-core is 1-block
- Core-periphery are (imperfect) 1-blocks
- Periphery-periphery is 0-block

Idealized Blockmodel

	CORE				PERIPHERY					
CORE	-	1	1	1	1	1	1	1	1	1
	1	-	1	1	1	1	1	1	1	1
	1	1	-	1	1	1	1	1	1	1
	1	1	1	-	1	1	1	1	1	1
PERIPHERY	1	1	1	1	-	0	0	0	0	0
	1	1	1	1	0	-	0	0	0	0
	1	1	1	1	0	0	-	0	0	0
	1	1	1	1	0	0	0	-	0	0
	1	1	1	1	0	0	0	0	-	0
	1	1	1	1	0	0	0	0	0	-

c_i = class (core or periphery) that node *i* is assigned to

$$\delta_{ij} = \begin{cases} 1 \; if \; c_i = CORE \; or \; c_j = CORE \\ 0 \; otherwise \end{cases}$$

Partitioning a Data Matrix

- Given a graphmatrix, we can randomly assign nodes to either core or periphery
- Search for partition that resembles the ideal

Assessing Fit to Data

a_{ij} = cell in data matrix

c_i = class (core or periphery) that node *i* is assigned to

$$\delta_{ij} = \begin{cases} 1 \ if \ c_i = CORE \ or \ c_j = CORE \\ 0 \ otherwise \end{cases}$$

$$\rho = \sum_{i,j} a_{ij} \delta_{ij}$$

- A Pearson correlation coefficient r(A,D) is

Alternative Images

	Core					Periphery				
C	-	1	1	1	1	0	0	0	0	0
o	1	-	1	1	1	0	0	0	0	0
r	1	1	-	1	1	0	0	0	0	0
e	1	1	1	-	1	0	0	0	0	0
	1	1	1	1	-	0	0	0	0	0
P	0	0	0	0	0	-	0	0	0	0
e	0	0	0	0	0	0	-	0	0	0
r	0	0	0	0	0	0	0	-	0	0
i	0	0	0	0	0	0	0	0	-	0
	0	0	0	0	0	0	0	0	0	-

Alternative Images

	Core					Periphery				
C	-	1	1	1	1	-	-	-	-	-
o	1	-	1	1	1	-	-	-	-	-
r	1	1	-	1	1	-	-	-	-	-
e	1	1	1	-	1	-	-	-	-	-
	1	1	1	1	-	-	-	-	-	-
P	-	-	-	-	-	-	0	0	0	0
e	-	-	-	-	-	0	-	0	0	0
r	-	-	-	-	-	0	0	-	0	0
I	-	-	-	-	-	0	0	0	-	0
	-	-	-	-	-	0	0	0	0	-

Continuous Model

- $X_{ij} \sim C_i C_j$
  - Strength or probability of tie between node i and node j is function of product of coreness of each
  - Central players are connected to each other
  - Peripheral players are connected only to core

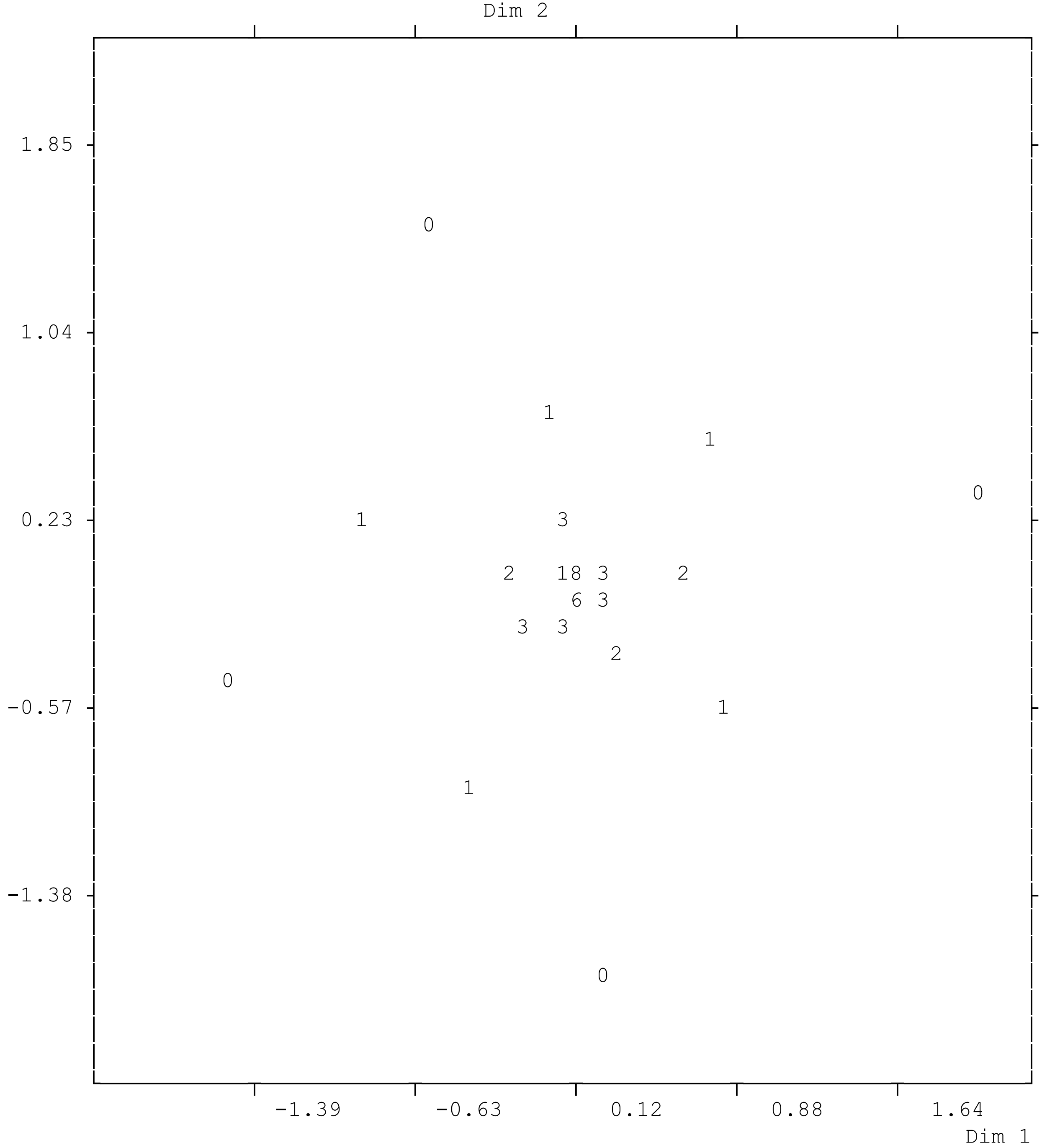
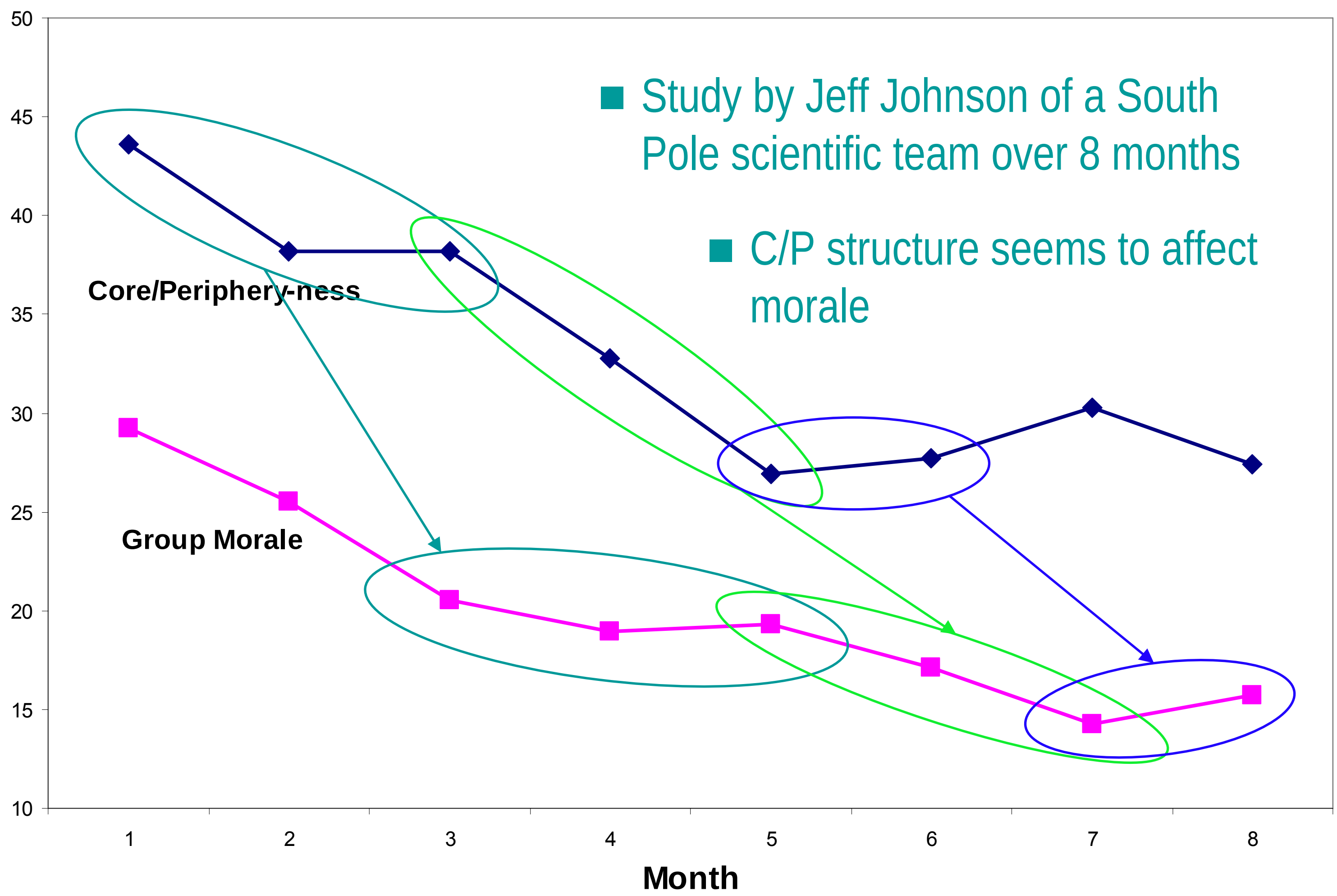


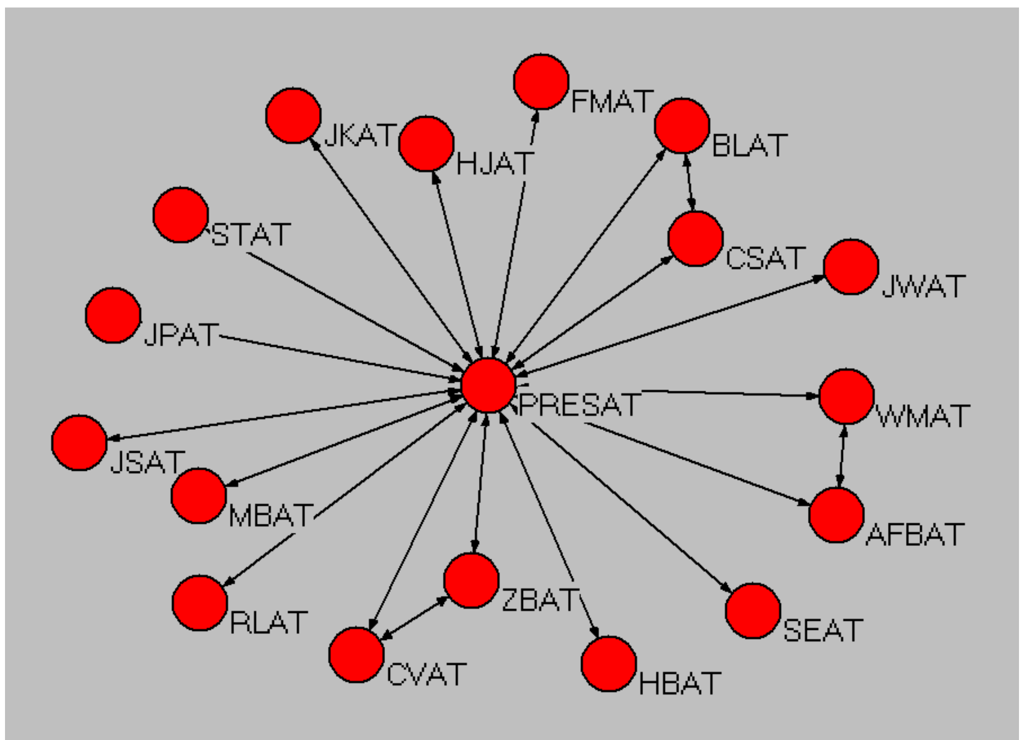
Figure 4. MDS of core/perip

CP Structures & Morale



Centralization

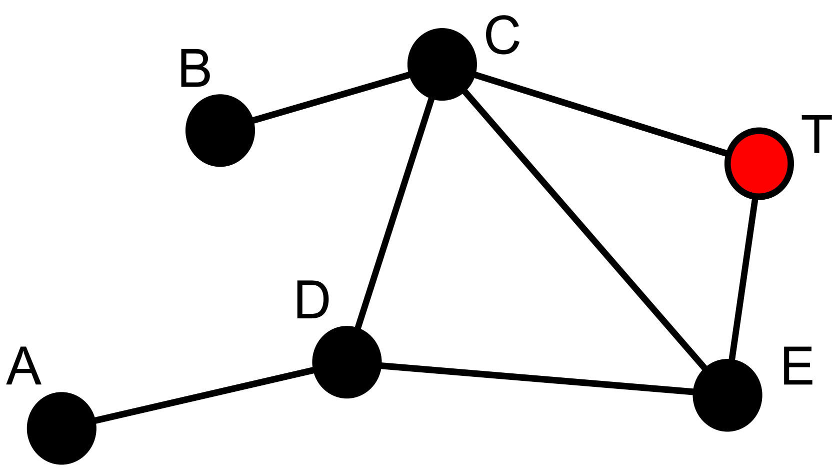
- Degree to which network revolves around a single node



Carter admin.
Year 1

Transitivity

- Proportion of triples with 3 ties as a proportion of triples with 2 or more ties
  - Aka the clustering coefficient



cc = 12/26 = 46.15%

{C,T,E} is a transitive triple, but {B,C,D} is not. {A,D,T} is not counted at all.

Dyadic Cohesion

- Adjacency ← Average is density
  - Strength of tie
- Distance
  - Length of shortest path between two nodes
- Multiplexity
  - Number of ties of different relations linking two nodes
- Number of paths linking two nodes

Classifying Cohesion