Structural Equivalence: Formalizing Position & Role

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The Dream

- Formalizing hallowed notions of position, role and structure
- Society as concrete network of relationships among individuals
  - And social structure is underlying network of positions structuring observed pattern among individuals
- Role freed from essentialist and culturalist definitions and defined in terms of characteristic relations among incumbents of positions, often reciprocally defined
  - Like functional role of species in ecosystem

Emergence

- If we can define roles formally based on observed relations, we can detect emergent, unnamed roles in groups

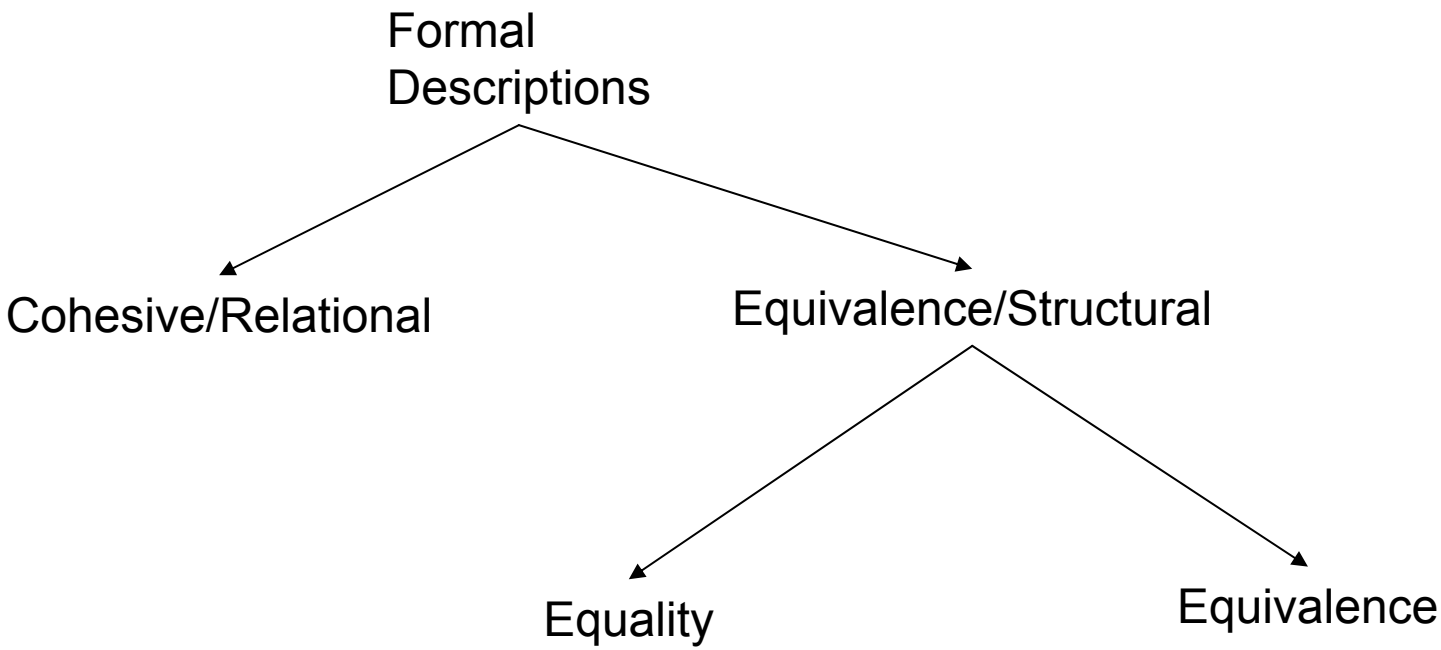
Cohesion vs Equivalence

- Connectionist vs structuralist approach

$$\frac{cohesion}{equivalence} :: \frac{proximity}{similarity} :: \frac{melody}{harmony} :: \frac{longitudinal}{\text{cross} - \text{sectional}} :: \frac{metonymy}{metaphor} :: \frac{complementarity}{competition}$$

Within Equivalence …

- Equality versus Isomorphism

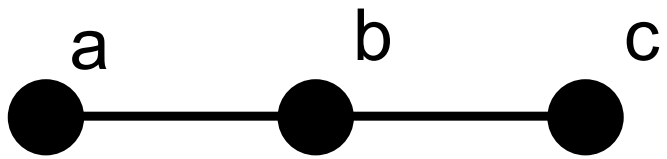


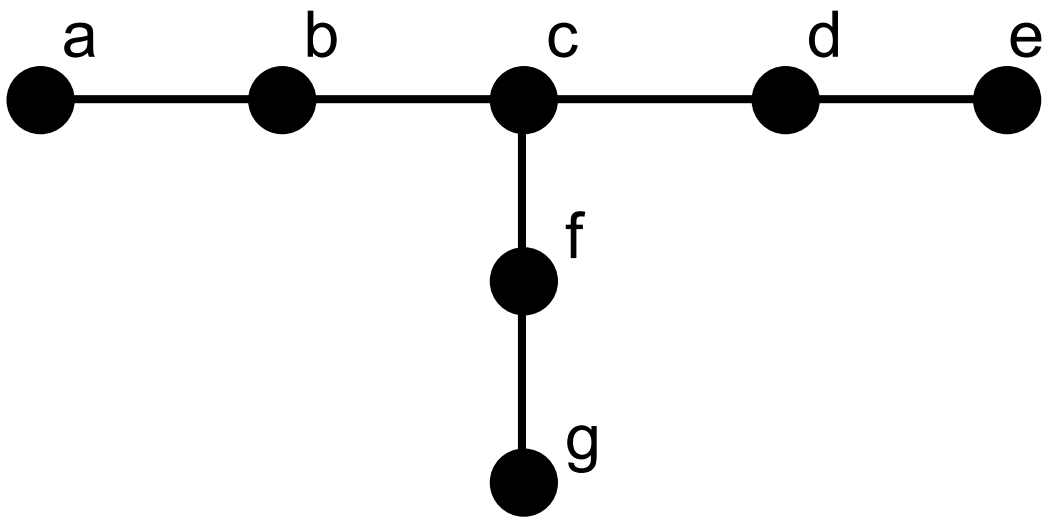
Positional Perspective

- Centrality measures one aspect of position
  - Unlike cohesive perspective, we cluster leaders with leaders, followers with followers, regardless of who they are tied to
- But there are other aspects
  - Not necessarily identified, nor summarizable in non-relational form

Experimental Exchange Nets

- Divvy up 24 points
- Who has what kinds of outcomes?





Whole Network Level

- Similarly, centralization classifies network shapes
  - Bavelas/Leavitt experiments

Implicit Hypothesis

- Similar nodes have similar outcomes
  - Occupy same position, then same results, even if unconnected to each other
- (Networks with similar structures will have similar outcomes)
  - Similarly structured teams will have similar performance outcomes

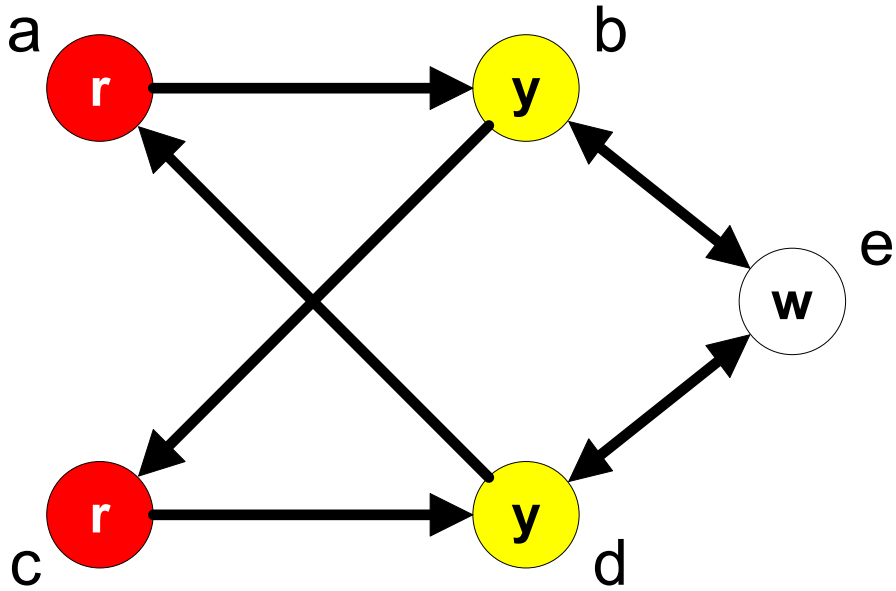
Generic Mechanisms

- Structural substitutability
  - Structural processes affect structurally similar nodes similarly
- Environmental determinism
  - Location, location, location
  - Nodes with similar environments are similarly affected by the environment
    - Important when environment is important
- (Imitation of similars)

Colorations

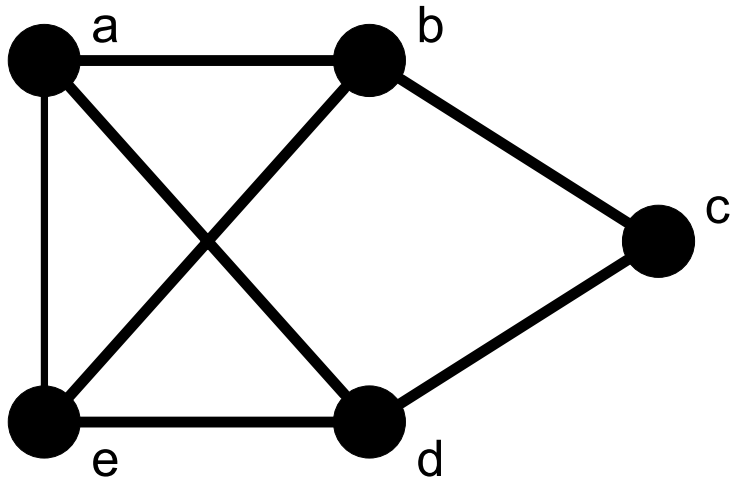
- A coloration C is just a partition of nodes.
  - Assignment of nodes to exhaustive, mutually exclusive classes
  - The color of a node *v*, written C(*v*) is just the equivalence class it belongs to
- An equivalence is just the relation E induced by a partition
- Is any relation that satisfies 3 conditions:
  - Transitive (a,b),(b,c) $\in$ E implies (a,c) $\in$ E
  - Symmetric (a,b) $\in$ E iff (b,a) $\in$ E
  - Reflexive (a,a) $\in$ E

Coloration



Neighborhoods

- Neighborhood of *v*, written N(*v*) is just the set of nodes adjacent to *v*.
- In digraphs, have
  - In-neighborhood $N^i(v)$: nodes sending arcs to *v*
  - Out-neighborhood $N^o(v)$: nodes receiving arcs from *v*
- Size of a node's neighbor-hood is just its degree

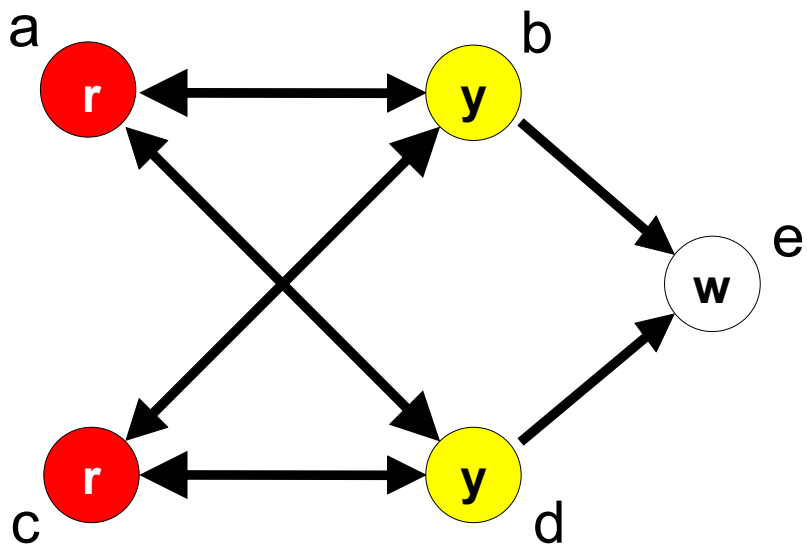


N(a) = {b,d,e}
N(c) = {b,d}

Definition of

Structural Equivalence

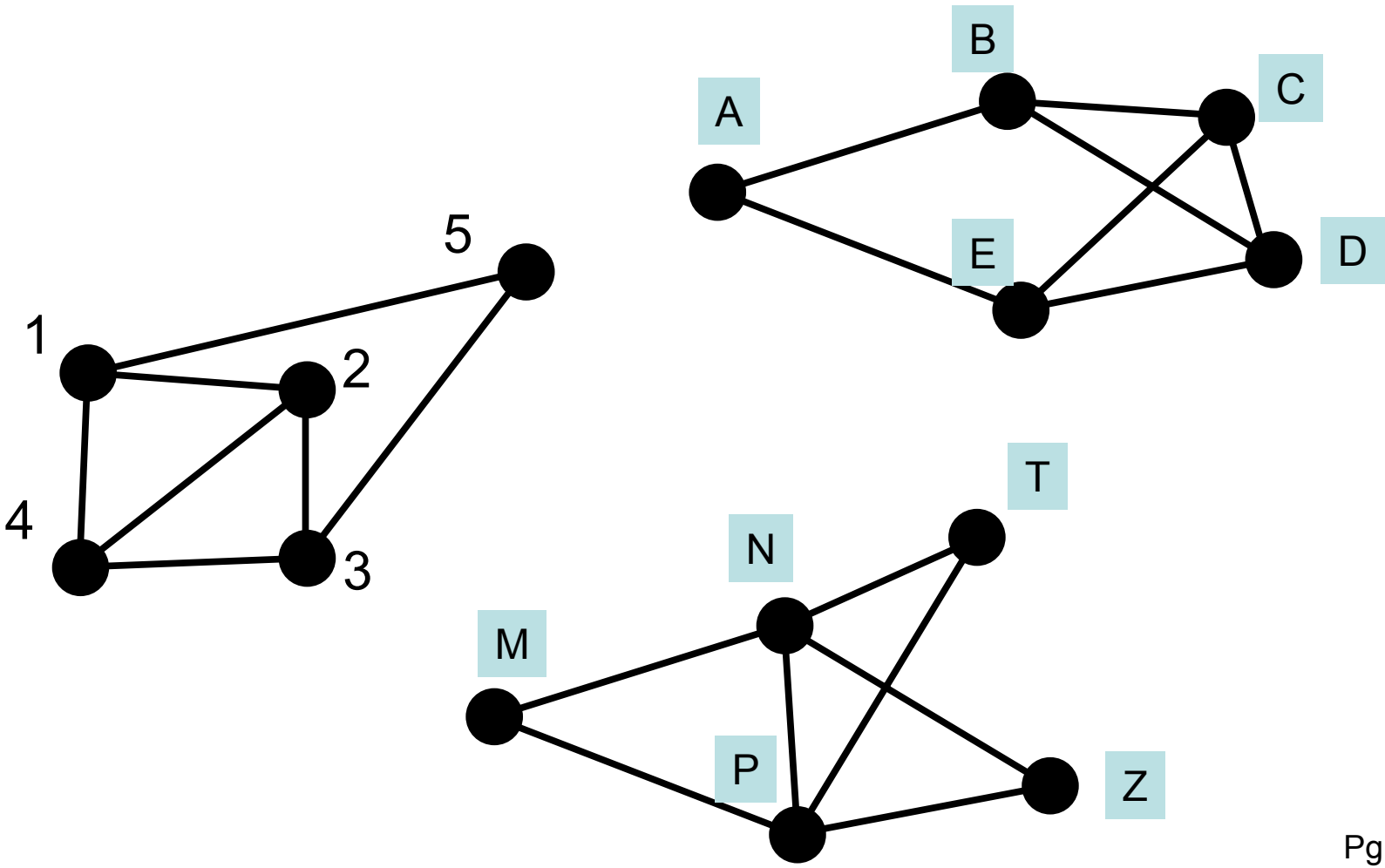
- A coloration C is <u>strongly</u> structural if C(u)=C(v) iff N(u)=N(v)
  - N(u) = N(v) iff $N^i(u)=N^i(v)$ and $N^o(u)=N^o(v)$



Isomorphism

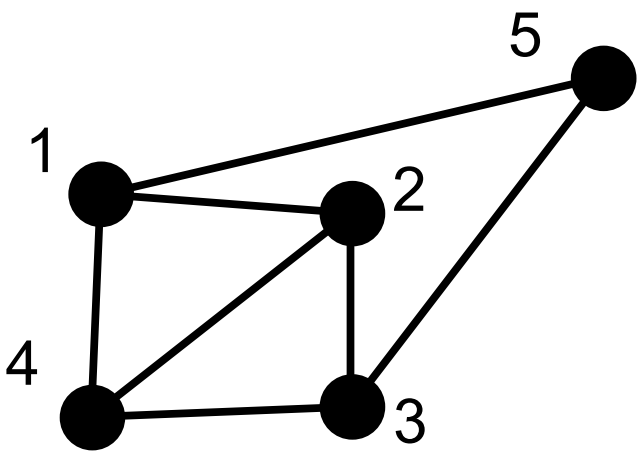
- Two graphs are isomorphic if you can find a 1:1 mapping of nodes of one to the other that preserves adjacency structure
- G(V,E) is isomorphic to G'(V',E') if there exists mapping p:V→V' such that (u,v) $\in$ E iff (p(u),p(v)) $\in$ E'
  - Such a mapping is called an isomorphism

Isomorphism



Automorphism

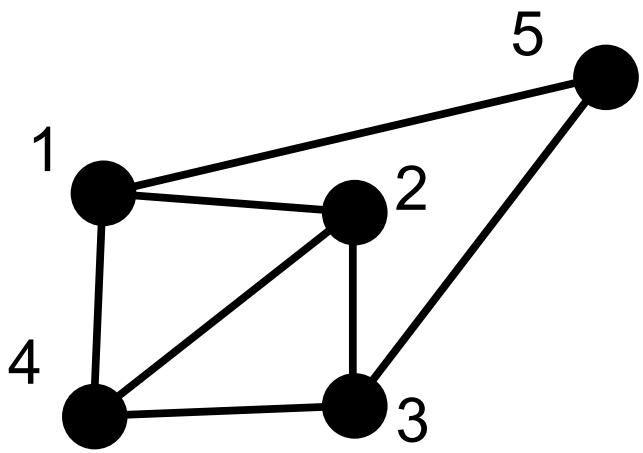
- Aka structural isomorphism
- An automorphism is an isomorphism of graph to itself



v	p(v)
1	3
2	4
3	1
4	2
5	5

Cycle Notation

- (1 3) (2 4) (5)



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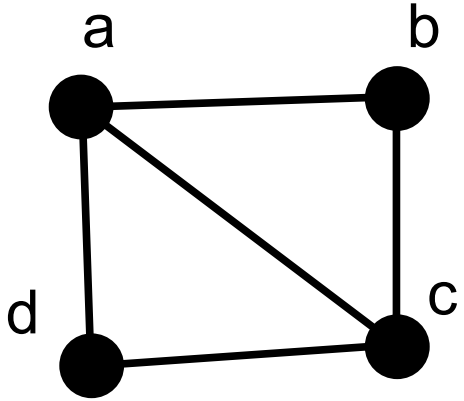
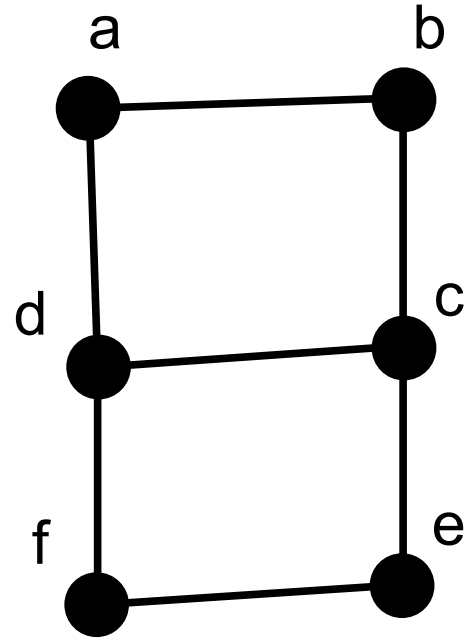
Automorphisms

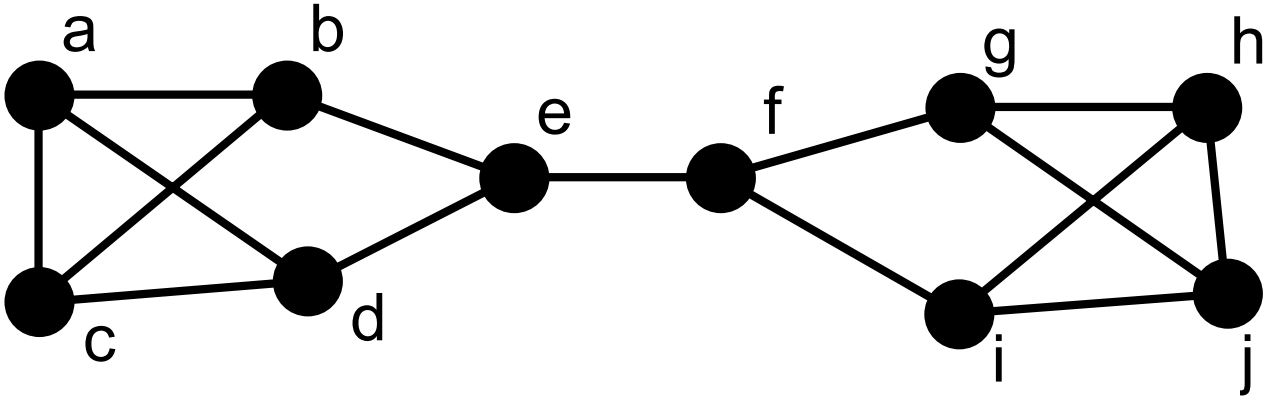




1. Is (a b) (d c) an automorphism?
2. Is (a d) (b c) an automorphism?
3. Is (a d) (b) (c) an automorphism?
4. Is (a c) (b d) an automorphism?

Weak Structural Equivalence

- A coloration C of G(V,E) is weakly structural if C(u)=C(v) iff the permutation p=(u v) is an automorphism of G

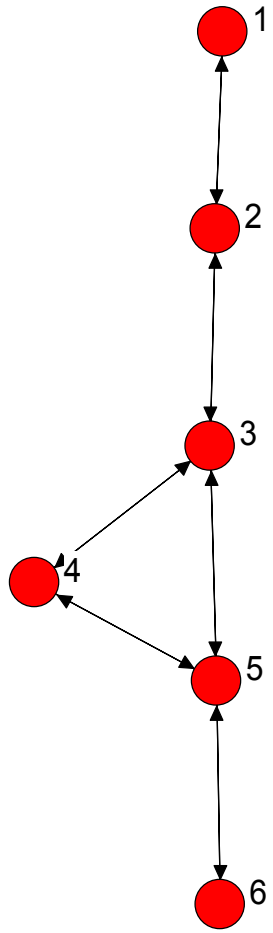


Structural Equivalence

- Structurally indistinguishable
  - Same degree, centrality, belong to same number of cliques, etc.
  - Only the label on the node can distinguish it from those equiv to it.
  - Perfectly substitutable: same contacts, resources
- Face the same social environment
  - Similar forces affecting them
  - On average, hear things equally early, influenced similarly, have similar things to cope with

S.E. as Model

- Captures notions like
  - Niche
    - Identical environments
  - Competition
    - For resources, attention, etc.
  - Location
    - What is location in a network if not who you are connected to?
  - Identity:
    - You are your friends

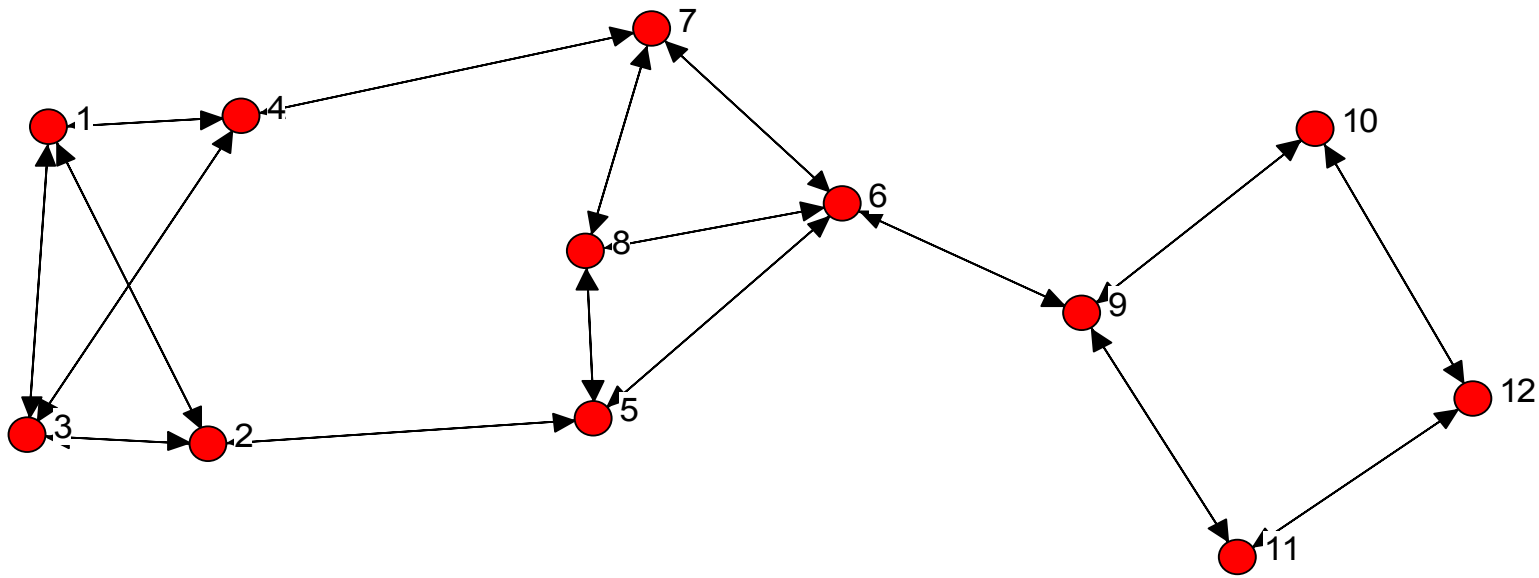


Issues with S.E.

- Theoretical mechanism
- Exchange experiments
- Social Role

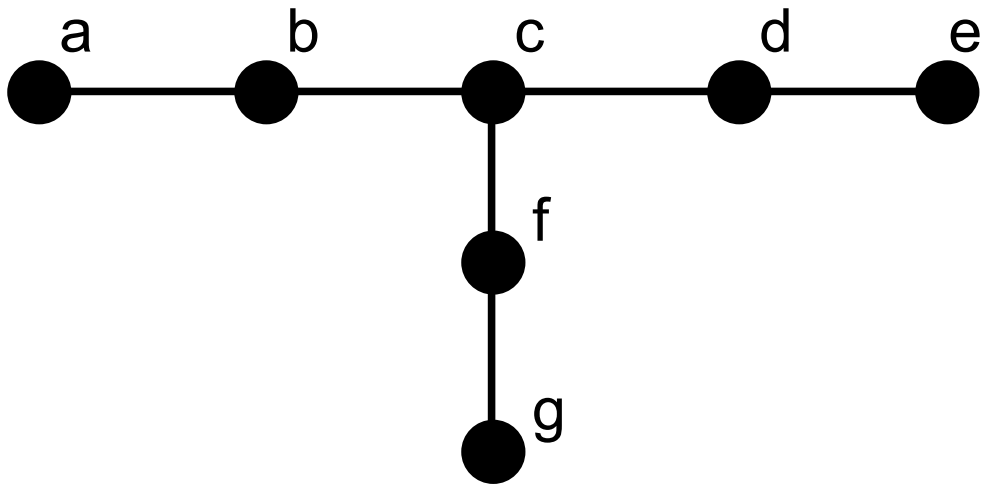
Issue #1: Mechanism

- What's the mechanism leading to similarity?
  - Confounds similarity with contiguity

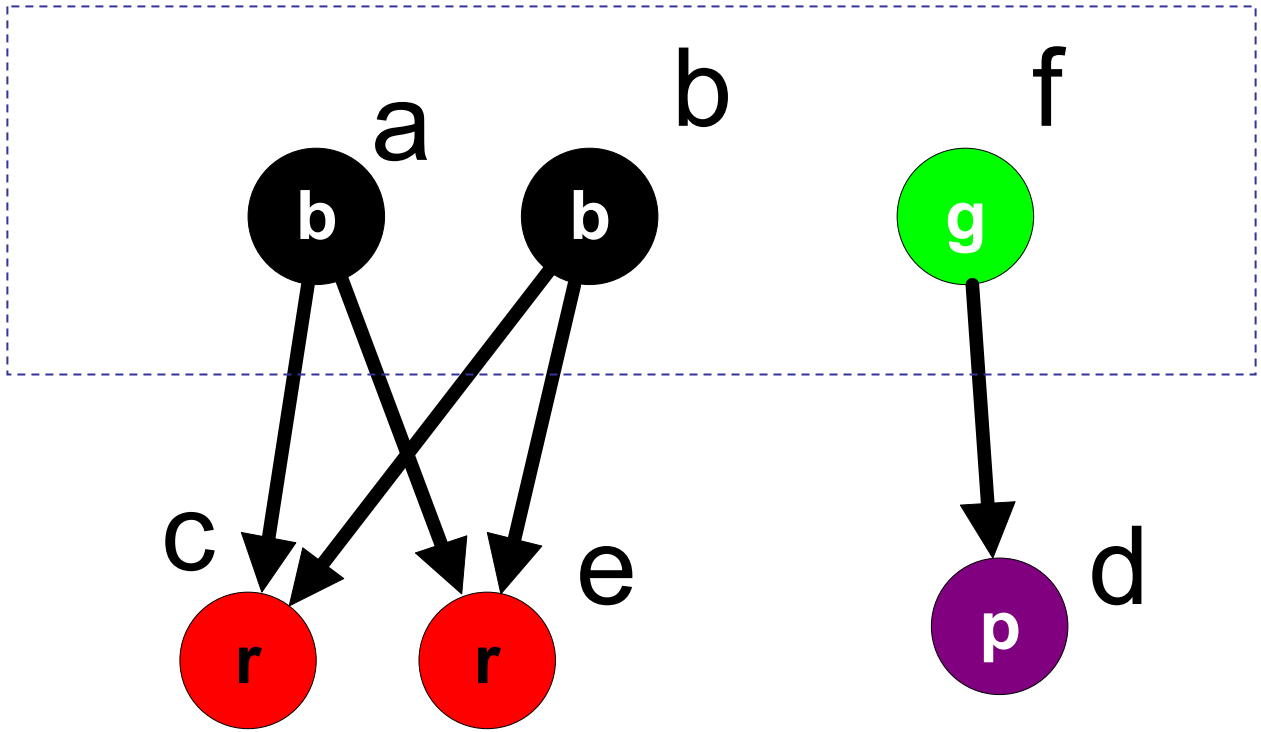


#2: Exchange Experiments

- Not helpful for explaining results of experimental exchange experiments



Issue #3: Social Role



Parents?

According to structural equivalence, only parents of the same children are playing the same role

Computation

- Relaxing concepts for real world data
- Two approaches
  - Discrete or blockmodel
    - Partition nodes into mutually exclusive classes such that departures from equivalence model are minimized
  - Profile similarity
    - For each pair of nodes, calculate the degree to which each pair is equivalent

Structural Equivalence

- Profile similarity method
  - Compute similarity/distance between rows of adjacency matrix
    - Correlation
    - Euclidean distance
  - Much argument over handling of diagonals
  - Can then MDS or cluster the resulting proximity matrix

Structural Equivalence

- Blockmodeling approach
  - Older Concor method
    - Actually based on profile method, but delivers discrete classes
  - Optimization method