Centrality

Structural Importance of Nodes

Life in the Military

A case by David Krackhardt

Roger was in charge of a prestigious Advisory Team, which made recommendations to the Joint Chiefs of Staff. His experience was considerable, and he was a well-respected authority in the area. Of the 16 people who worked for him, he trusted those who also had a considerable amount of wartime experience, either in Vietnam or in other combat operations. He found their counsel to be particularly valuable.

Life in the Military

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Roger and Rick each had a PhD, and the remaining people all had graduate professional degrees in a variety of areas. Bob, Pete, Red and Sally were the newest members of the Team (they had been there for almost a year), and were fresh out of training in advanced weapons technology. Pete was the youngest member of the team. His background was computer science, and he had worked at MIT in their Draper Labs on simulations of war strategies using various weaponry.

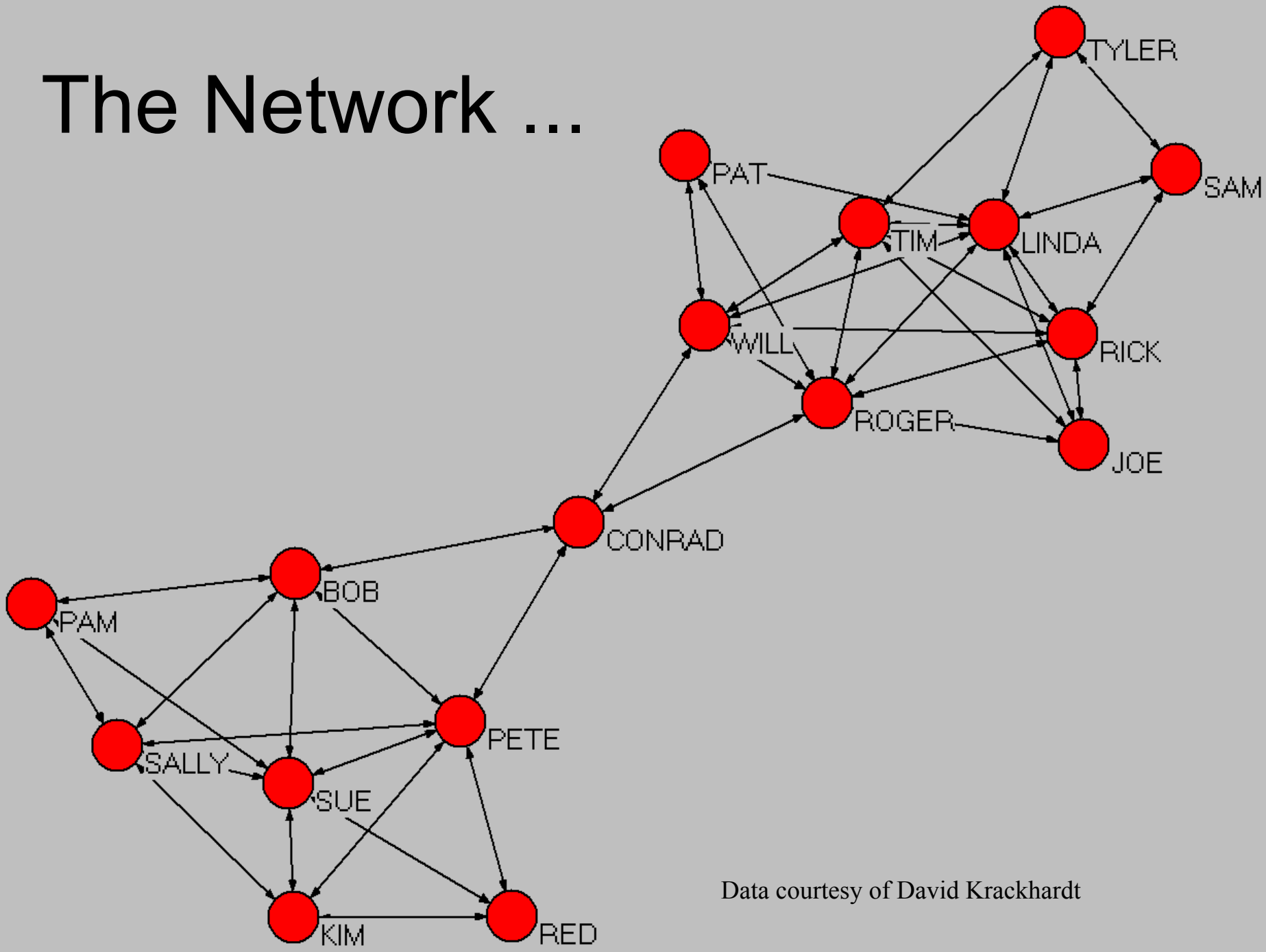
Life in the Military … cont.

Linda was a senior member of the team and also one of the most approachable. She saw it as part of her responsibility to make sure people were getting along with each other, since cooperation across this disparate group was critical to its effectiveness. She and Rick would frequently hold social events to help solidify the group. Linda had been with the group the longest (almost 12 years) and had seen it grow in stature and respect over that time.

Life in the Military … cont.

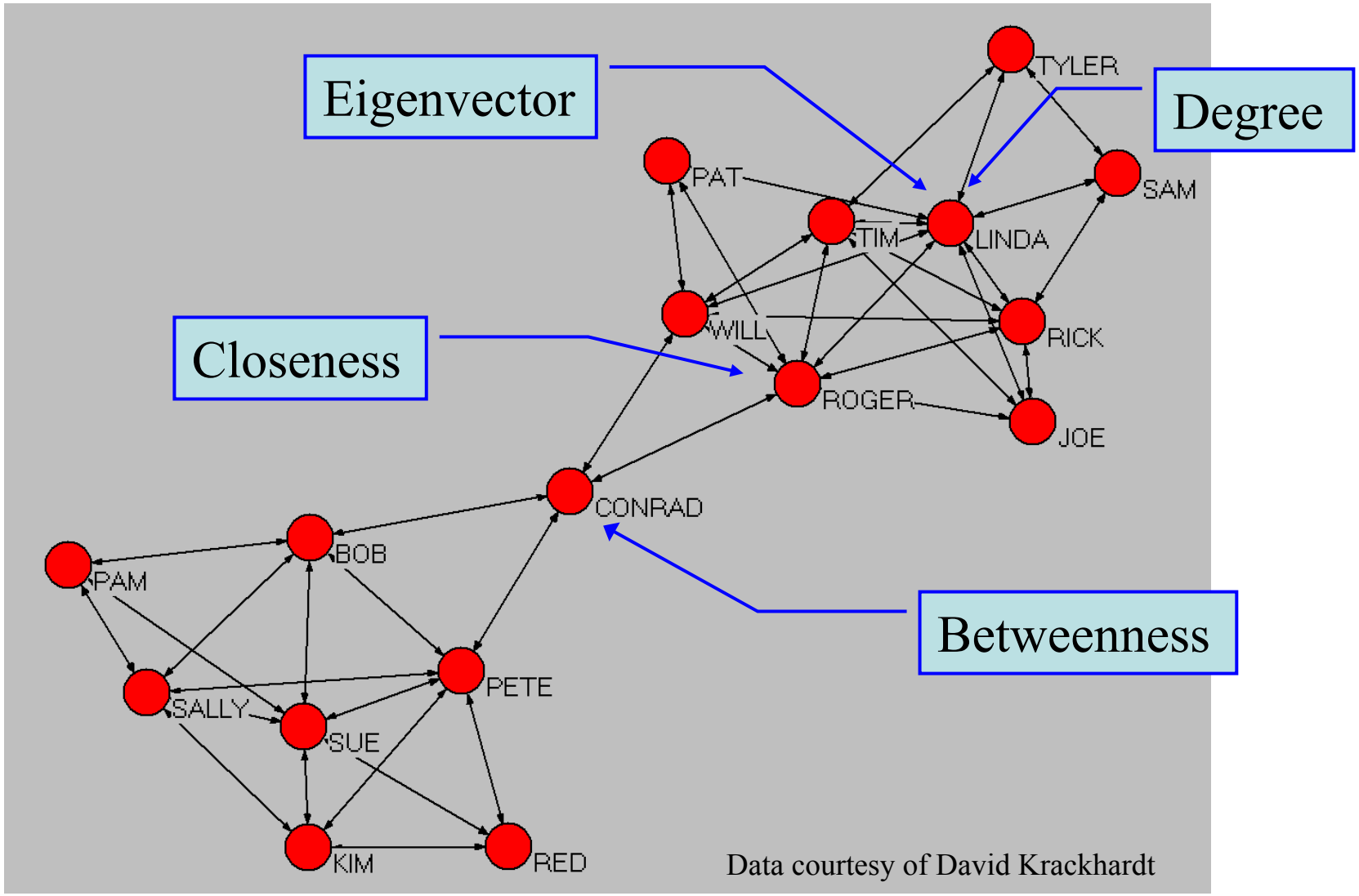
Roger had been criticized recently for his management style, which was admittedly authoritarian. At the request of some of his colleagues, he had called in an organizations consultant to advise him and the Team how to best proceed with teamwork and other managerial issues. The consultant ran team-building workshops. Roger felt that the consultant was a “touchy-feely” type and that the experience had been a total waste of time. He refused to bring in any more consultants. Some of the Team members were talking behind the scenes about resigning or requesting a transfer.

The Network ...



Data courtesy of David Krackhardt

Four Aspects of Centrality



Degree Centrality

- Number of ties that involve a given node
  - Marginals of symmetric adjacency matrix

	I1	I3	W1	W2	W3	W4	W5	W6	W7	W8	W9	S1	S2	S4	Deg
I1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	4
I3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
W1	1	0	0	1	1	1	1	0	0	0	0	1	0	0	6
W2	1	0	1	0	1	1	0	0	0	0	0	1	0	0	5
W3	1	0	1	1	0	1	1	0	0	0	0	1	0	0	6
W4	1	0	1	1	1	0	1	0	0	0	0	1	0	0	6
W5	0	0	1	0	1	1	0	0	1	0	0	1	0	0	5
W6	0	0	0	0	0	0	0	0	1	1	1	0	0	0	3
W7	0	0	0	0	0	0	1	1	0	1	1	0	0	1	5
W8	0	0	0	0	0	0	0	1	1	0	1	0	0	1	4
W9	0	0	0	0	0	0	0	1	1	1	0	0	0	1	4
S1	0	0	1	1	1	1	1	0	0	0	0	0	0	0	5
S2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S4	0	0	0	0	0	0	0	0	1	1	1	0	0	0	3

Degree Centrality

- Index of exposure to what is flowing through the network
  - Gossip network: central actor more likely to hear a given bit of gossip
- Interpreted as opportunity to influence & be influenced directly
- Predicts variety of outcomes from virus resistance to power & leadership to job satisfaction to knowledge

Closeness Centrality

- Sum of distances to all other nodes
  - Computed as marginals of symmetric geodesic distance matrix

	I1	I3	W1	W2	W3	W4	W5	W6	W7	W8	W9	S1	S2	S4	Clo
I1	0	?	1	1	1	1	2	4	3	4	4	2	?	4	27
I3	?	0	?	?	?	?	?	?	?	?	?	?	?	?	0
W1	1	?	0	1	1	1	1	3	2	3	3	1	?	3	20
W2	1	?	1	0	1	1	2	4	3	4	4	1	?	4	26
W3	1	?	1	1	0	1	1	3	2	3	3	1	?	3	20
W4	1	?	1	1	1	0	1	3	2	3	3	1	?	3	20
W5	2	?	1	2	1	1	0	2	1	2	2	1	?	2	17
W6	4	?	3	4	3	3	2	0	1	1	1	3	?	2	27
W7	3	?	2	3	2	2	1	1	0	1	1	2	?	1	19
W8	4	?	3	4	3	3	2	1	1	0	1	3	?	1	26
W9	4	?	3	4	3	3	2	1	1	1	0	3	?	1	26
S1	2	?	1	1	1	1	1	3	2	3	3	0	?	3	21
S2	?	?	?	?	?	?	?	?	?	?	?	?	0	?	0
S4	4	?	3	4	3	3	2	2	1	1	1	3	?	0	27

Closeness Centrality

- Is an inverse measure of centrality
- Index of expected time until arrival for given node of whatever is flowing through the network
  - Gossip network: central player hears things first

Betweenness Centrality

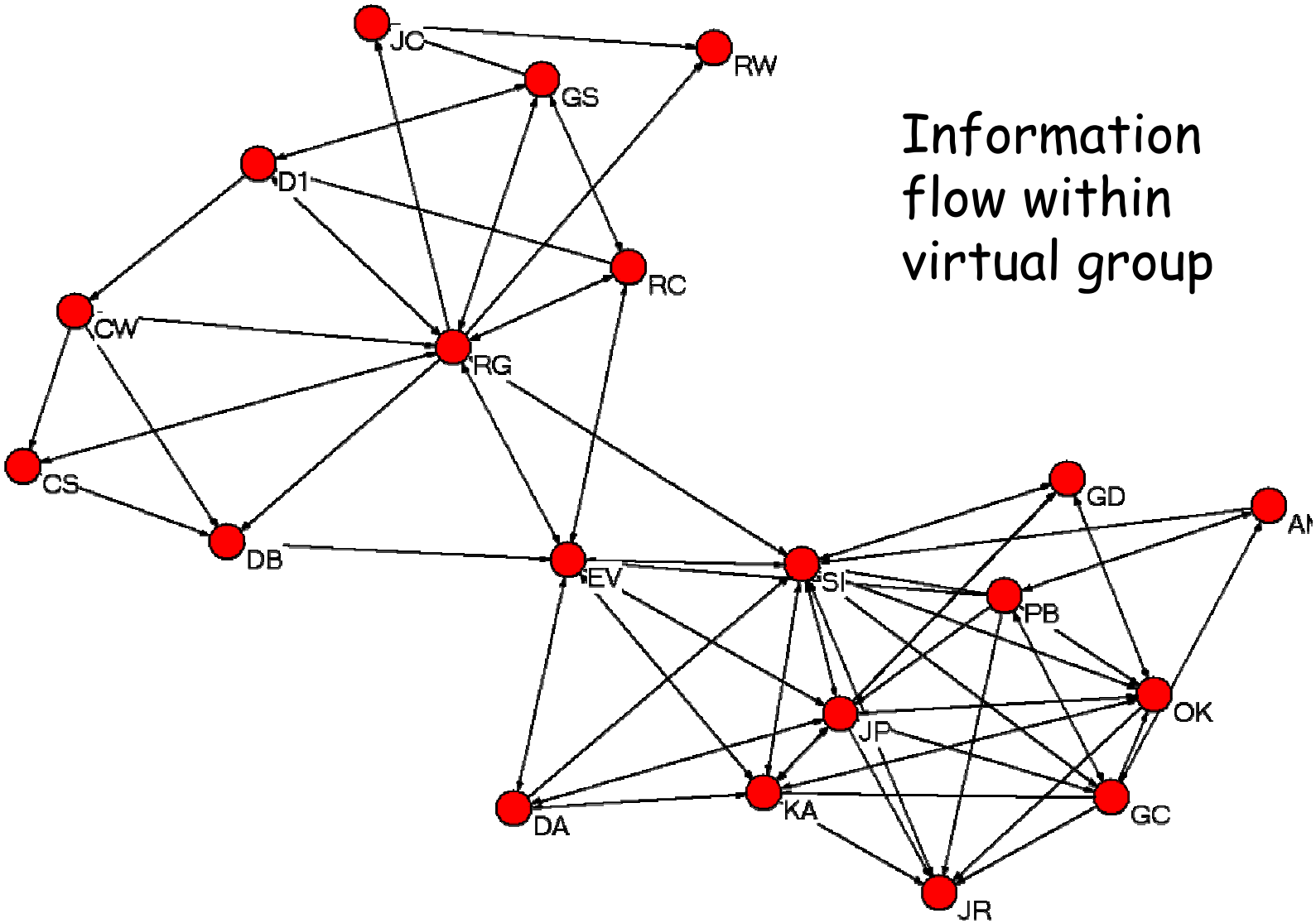
- How often a node lies along the shortest path between two other nodes
  - Computed as:

$$b_k = \sum_{i,j} \frac{g_{ikj}}{g_{ij}}$$

where gij is number of geodesic paths from i to j and gikj is number of those paths that pass through k

- Index of potential for gatekeeping, brokering, controlling the flow, and also of liaising otherwise separate parts of the network;
- Interpreted as indicating power and access to diversity of what flows; potential for synthesizing

Local Gain is Global Pain

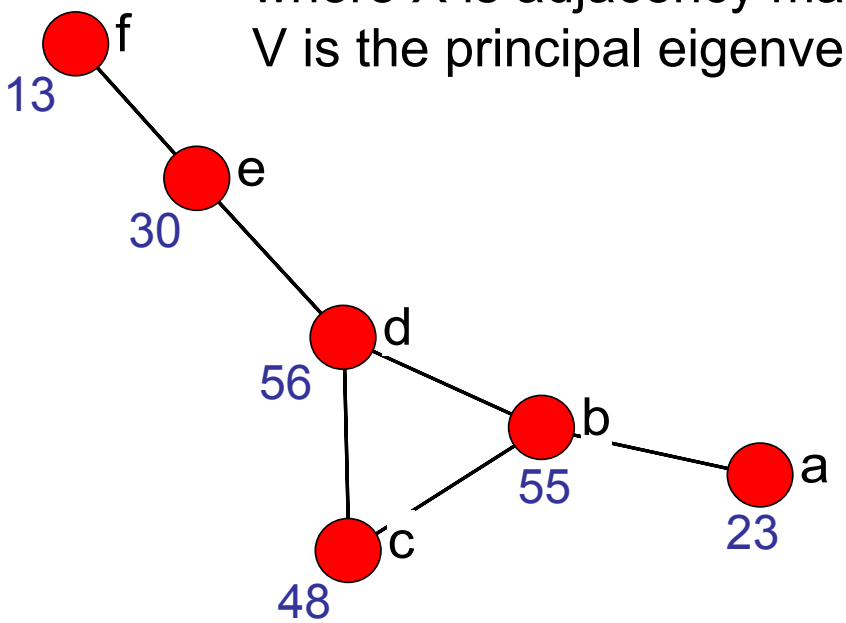


Cross, Parker, & Borgatti, 2002. Making Invisible Work Visible. *California Management Review*. 44(2): 25-46

Eigenvector Centrality

- Node has high score if connected to many nodes are themselves well connected
  - Computed as: $\lambda v = Av$

where X is adjacency matrix and V is eigenvector centrality. V is the principal eigenvector of X.



	a	b	c	d	e	f	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
a	0	1	0	0	0	0	1	3	6	16	35	86	195	465	1071	2524
b	1	0	1	1	0	0	3	6	16	35	86	195	465	1071	2524	5854
c	0	1	0	1	0	0	2	6	13	32	73	173	401	940	2190	5117
d	0	1	1	0	1	0	3	7	16	38	87	206	475	1119	2593	6086
e	0	0	0	1	0	1	2	4	9	20	47	107	253	582	1372	3175
f	0	0	0	0	1	0	1	2	4	9	20	47	107	253	582	1372

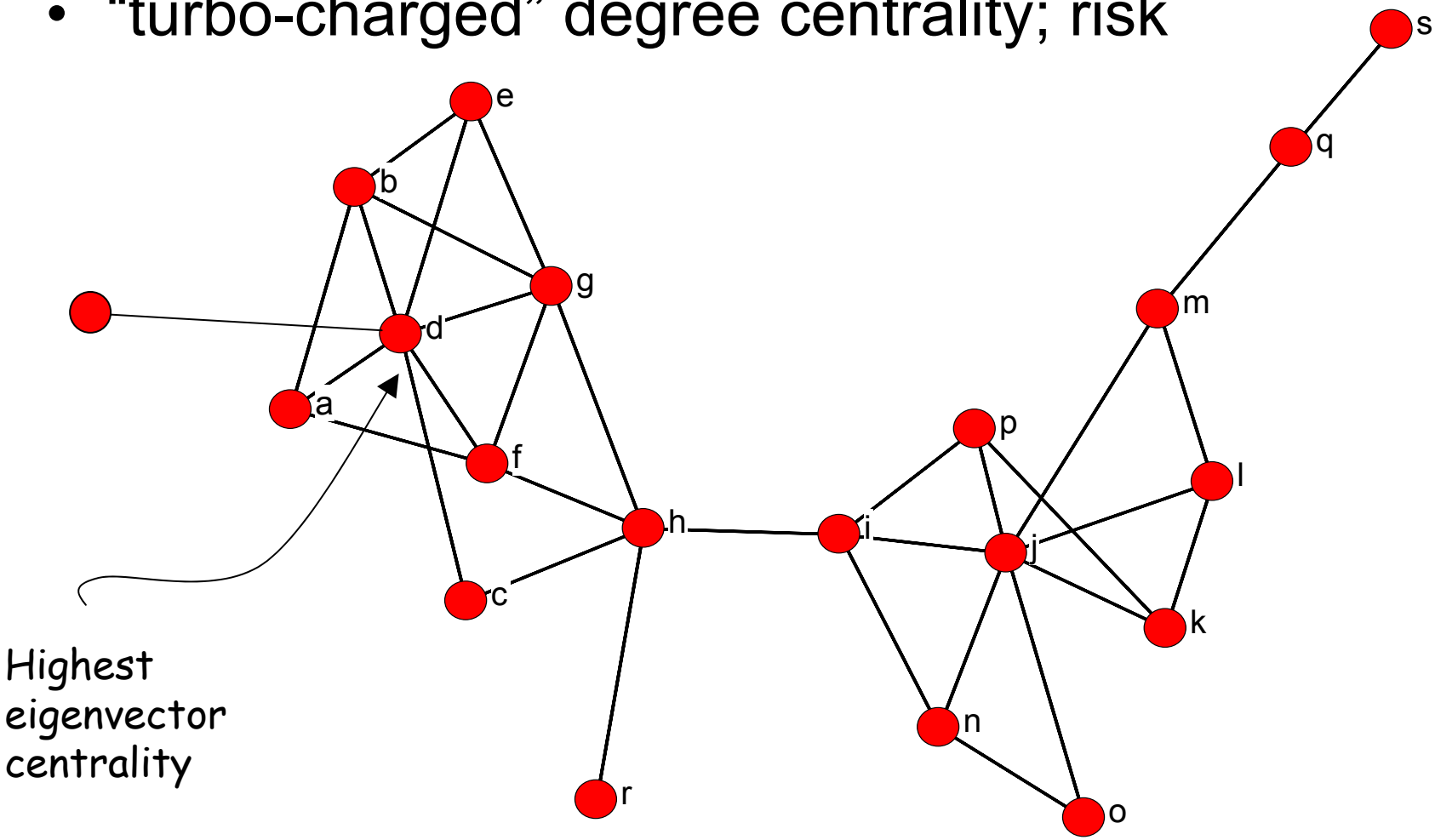
A	8.3	10.7	9.4	10.7	10.1	10.6	10.3	10.5	10.4	10.5
B	25.0	21.4	25.0	23.3	24.7	24.0	24.5	24.2	24.4	24.3
C	16.7	21.4	20.3	21.3	21.0	21.3	21.1	21.2	21.2	21.2
D	25.0	25.0	25.0	25.3	25.0	25.3	25.1	25.3	25.1	25.2
E	16.7	14.3	14.1	13.3	13.5	13.1	13.3	13.1	13.3	13.2
F	8.3	7.1	6.3	6.0	5.7	5.8	5.6	5.7	5.6	5.7

Eigenvector Centrality

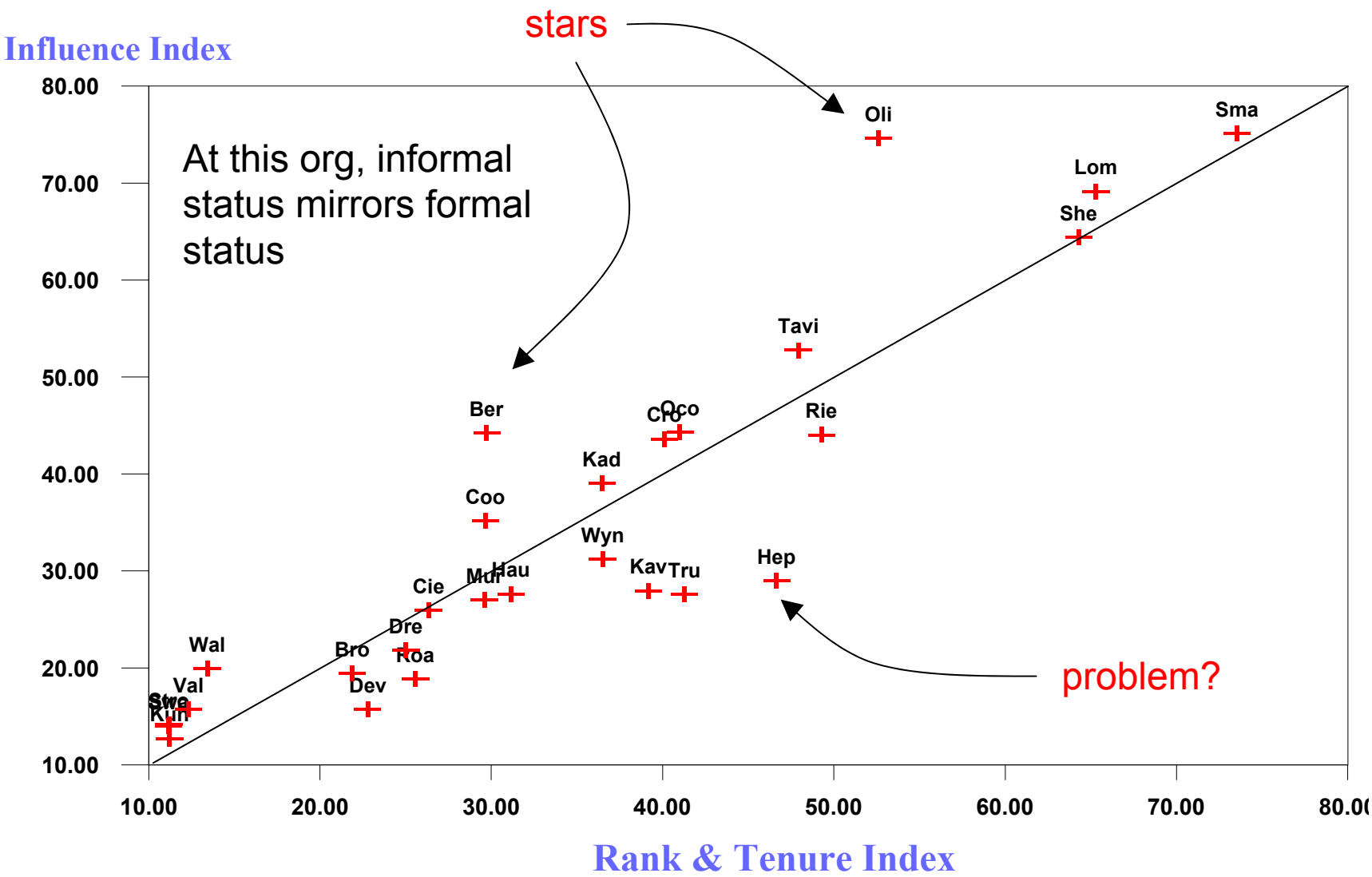
- Indicator of popularity,
  - "in the thick of things"
- Like degree, is index of exposure, risk
- Tends to identify centers of large cliques

Eigenvector Centrality

- "turbo-charged" degree centrality; risk



Influence Network



Walk-Based Measures

- Multiple motivations
  - actor's status is function of not only the number of people who choose them, but their status
  - in an influence process, an actor's impact on another is function of all sequences (walks) linking them
- Resulting measures are similar / related

Influence Approach

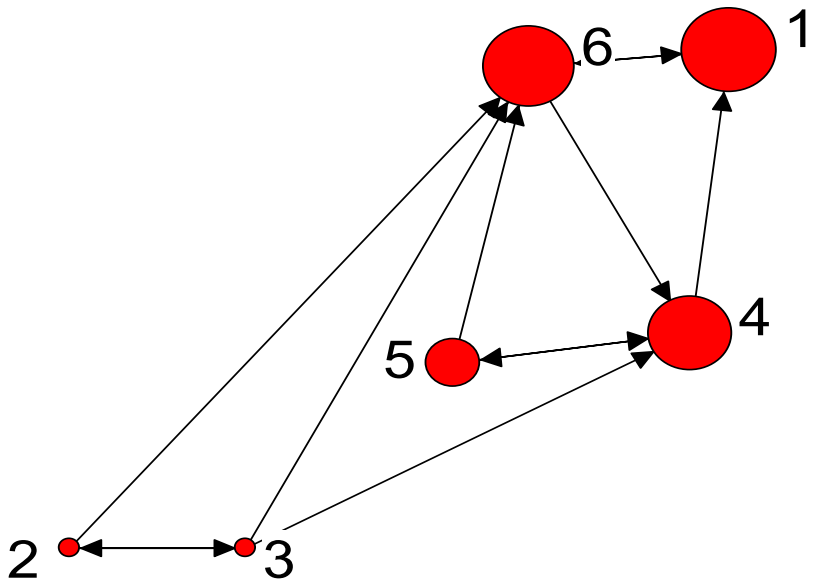
- Variations by Katz, Friedkin, Taylor, etc.
- Generic approach
  - R is network matrix, α is attenuation parameter
  - $\mathbf{Q} = \alpha^0\mathbf{R}^0 + \alpha^1\mathbf{R}^1 + \alpha^2\mathbf{R}^2 + \alpha^3\mathbf{R}^3 + \ldots \alpha^\infty\mathbf{R}^\infty$
  - $\mathbf{Q} = (\mathbf{I}-\alpha\mathbf{R})^{-1}$, assuming $\alpha^{-1} > \lambda_1$
  - $\mathbf{s} = (\mathbf{I}-\alpha\mathbf{R})^{-1}\mathbf{1} = \mathbf{Q1}$ (row sums of **Q**)

Recursive Status Approach

- Hubbell
  - **s** = **Ws** + **e**, where **W** is adj matrix w/ equal col sums < 1, **s** is vector representing status, **e** is vector of exogeneous inputs (usually 1s)
  - $\mathbf{s} = (\mathbf{I}-\mathbf{W})^{-1}\mathbf{e}$
- Bonacich, Coleman, Burt, etc.
  - Principal eigenvector of **W**
  - $\lambda\mathbf{c} = \mathbf{Wc}$ (or **W'c** if appropriate)

Katz example

Who really knows what's going on?



Node	Out	In	Katz
1	1	2	13.0
2	2	1	1.0
3	3	1	1.0
4	2	3	11.4
5	2	1	6.2
6	2	4	12.6

Indegree gives same score to 5 as to 2 and 3. But 5 is chosen by 4, who is chosen by popular nodes like 6. Katz score gives 5 much higher score than 2 or 3. Similarly node 1 has only two incoming choices, but they are from the most sought-after players, so 1 must be even more knowledgeable than they.

Centrality

- Structural importance
- Many measures
  - very different assumptions about data, processes & objectives
- Basically count paths or walks
  - emanating from / terminating at given node
  - passing through a given node