Equivalence

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MB 874 Social Network Analysis

The Dream

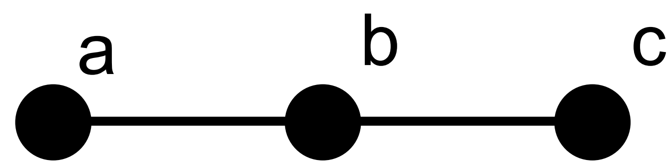
- Formalizing hallowed notions of position, role and structure
- Society as concrete network of relationships among individuals
  - And social structure is underlying network of positions structuring observed pattern among individuals
- Role freed from essentialist and culturalist definitions and defined in terms of characteristic relations among incumbents of positions, often reciprocally defined
  - Like functional role of species in ecosystem

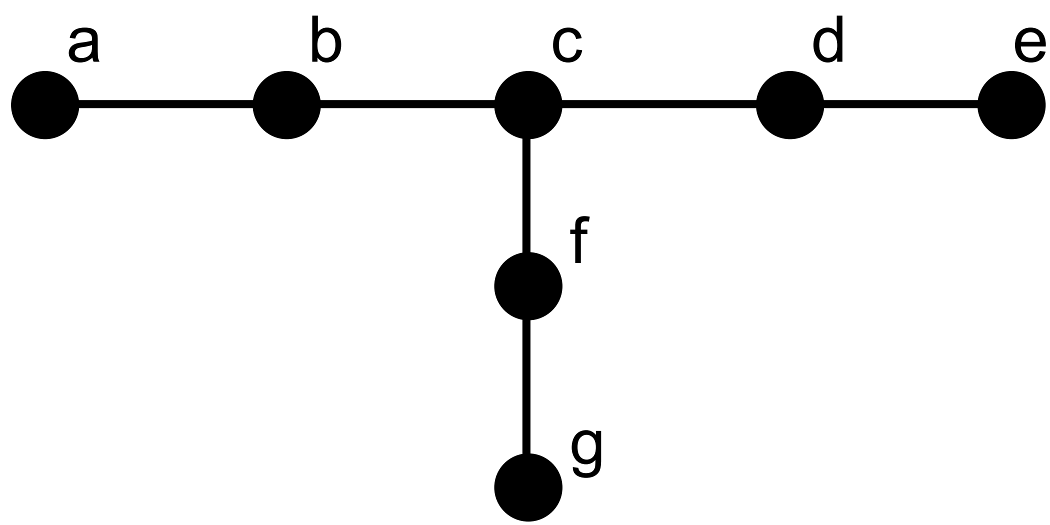
Positional Perspective

- Centrality measures one aspect of position
  - Unlike cohesive perspective, we class leaders with leaders, followers with followers, regardless of who they are tied to
- But there are other aspects
  - Not necessarily identified, nor summarizable in non-relational form

Experimental Exchange Nets

- Divvy up 24 points
- Who has what kinds of outcomes?





Implicit Hypothesis

- Similar nodes have similar outcomes
  – Occupy same position, then same results
- (Networks with similar structures will also have similar outcomes)
  – Similarly structured teams will have similar performance outcomes

Emergence

- If we can define roles formally based on observed relations, we can detect emergent, unnamed roles in groups

Cohesion vs Equivalence

- Connectionist vs structuralist approach

$$\frac{cohesion}{equivalence} :: \frac{proximity}{similarity} :: \frac{melody}{harmony} :: \frac{longitudinal}{\text{cross}-\text{sectional}} :: \frac{metonymy}{metaphor} :: \frac{complementarity}{competition}$$

A Collection of Concepts

- Structural equivalence
- Automorphic equivalence
- Maximal regular equivalence
- Notes
  - Lattice of regular equivalences
  - Equivalences versus colorations (partitions)

Agenda

- Three equivalence concepts from theoretical point of view
- Computation and implementation

Structural Equivalence

Colorations

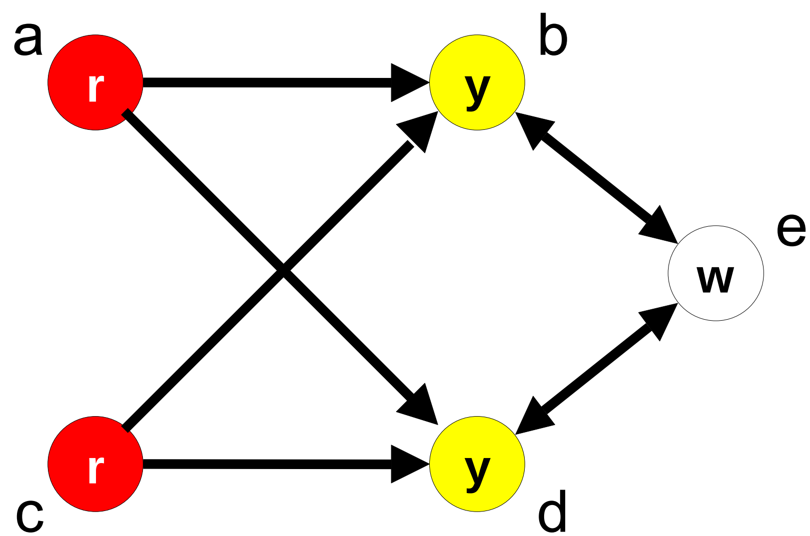
- A coloration C is just a partition of nodes.
  - Assignment of nodes to exhaustive, mutually exclusive classes
  - The color of a node *v*, written C(*v*) is just the equivalence class it belongs to
- An equivalence is just the relation E induced by a partition
- Is any relation that satisfies 3 conditions:
  - Transitivity: (a,b), (b,c) $\in$ E implies (a,c) $\in$ E
  - Symmetricity: (a,b) $\in$ E iff (b,a) $\in$ E
  - Reflexivity: (a,a) $\in$ E

Image Graphs

- Simplified models of a network, usually with a set of rules that describe correspondence between network and model

Structural Equivalence (simplified definition)

- u ≡ v if, for any w, whenever u→w then v→w, and whenever w→u then w→v



Note: Equivalent nodes have been colored the same.

- C(u) = C(v) if N(u) = N(v)
- C(u) = C(v) if $N^{out}(u) = N^{out}(v)$ and $N^{in}(u) = N^{in}(v)$

Structural Equivalence

- Structurally indistinguishable
  - Same degree, centrality, belong to same number of cliques, etc.
  - Only the label on the node can distinguish it from those equiv to it.
  - Perfectly substitutable: same contacts, resources
- Face the same social environment
  - Similar forces affecting them

Classical Hypothesis

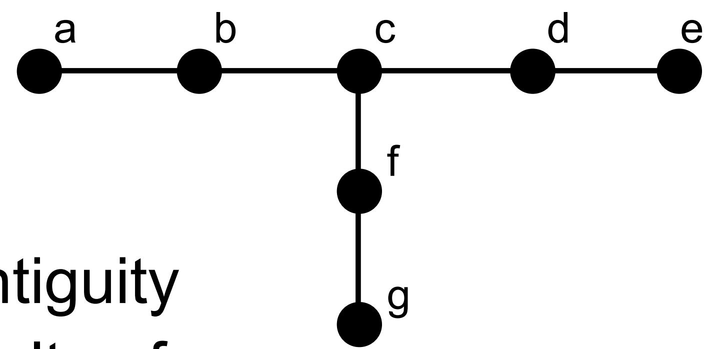
- Structurally equivalent nodes will have similar internal structures | attitudes | outcomes
  - i.e., an explanation for homogeneity

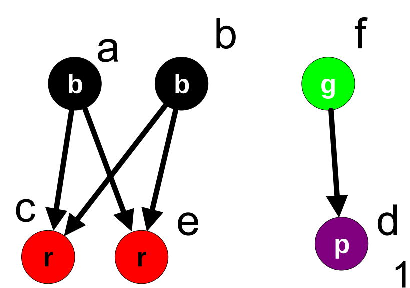
Mechanisms of Homogeneity

- Structural indistinguishability in the context of structural processes
  - Centrality
  - Structural holes
- Similar responses to similar environment
  - adaptation
- Diffusion
  - Through common third parties

Pros and Cons of SE

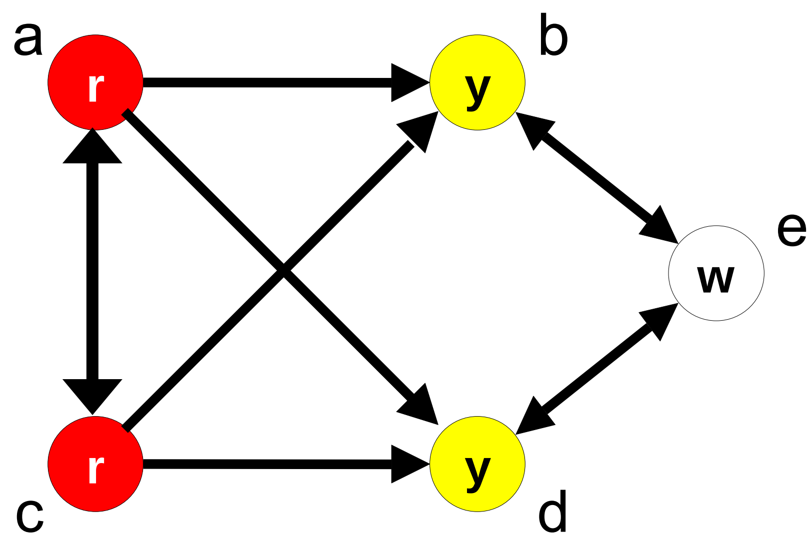
- Pros
  - Captures notions like niche
  - Location or position
    - You are your friends
- Cons
  - Confounds similarity with contiguity
  - Not helpful for explaining results of exchange experiments
  - Not a good formalization of social role
    - Mother & father play same role to their kids, but not other parents
    - Can't use in disconnected graphs





Technicality

- Definition "fails" when structurally equivalent nodes are tied to each other

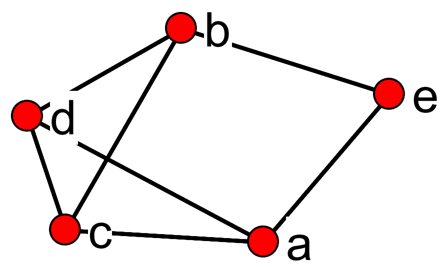


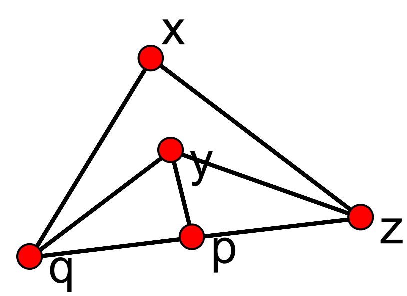
- C(u) = C(v) if N(u)-{v} = N(v)-{u} is better*

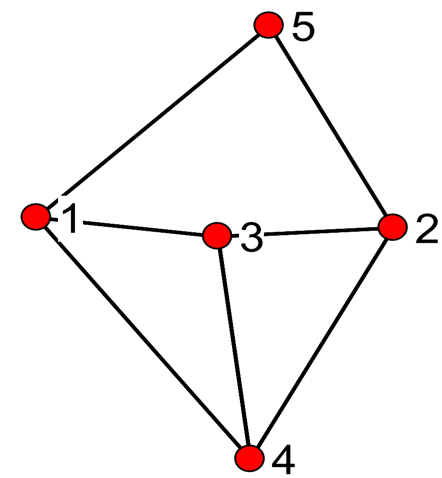
*Even better definition is available but is more advanced

Automorphic Equivalence

Isomorphisms







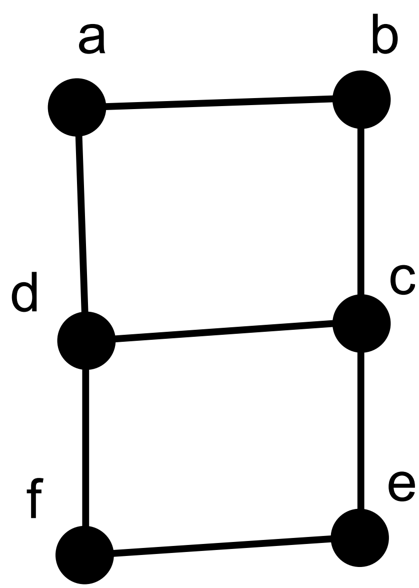
Mappings:

Fig 1	Fig 2	Fig 3
a	q	1
b	z	2
c	y	3
d	p	4
e	x	5

A mapping *p* from one graph to another is an isomorphism if whenever u is tied to v, p(u) is tied to p(v).

Isomorphisms are mappings that preserve structure

Automorphism



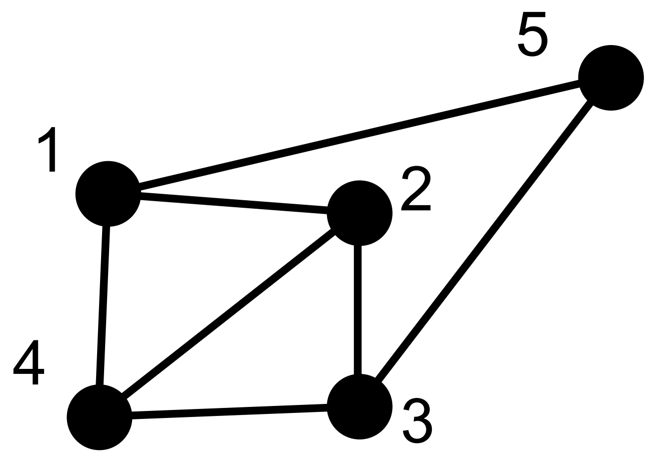
An isomorphism from one graph to the same graph is an automorphism

G	P(G)	P'(G)
a	B	F
b	A	E
c	D	C
d	C	D
e	F	B
f	E	A

Automorphisms constitute the "symmetries" of a graph.

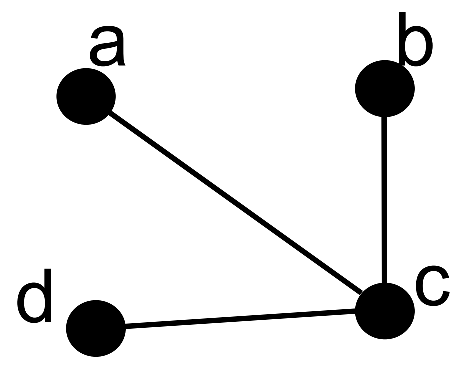
Cycle Notation

- (1 3) (2 4) (5)



v	p(v)
1	3
2	4
3	1
4	2
5	5

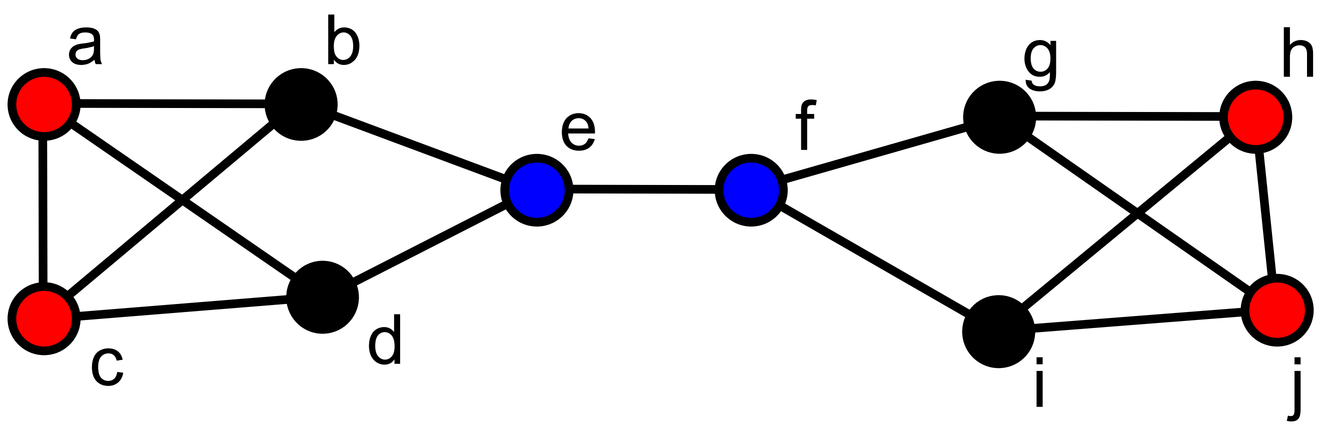
- (a b d) (c)



v	p(v)
a	b
b	d
c	c
d	a

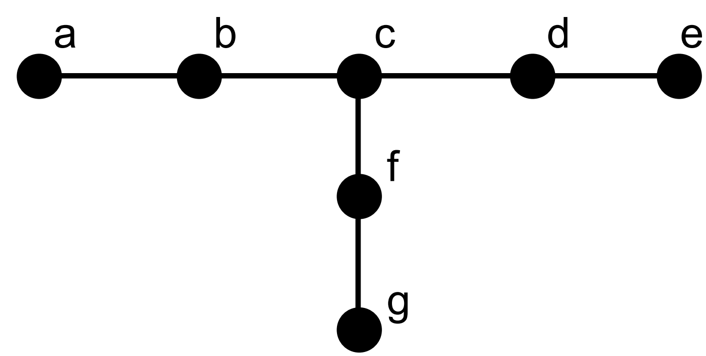
Automorphic Equivalence

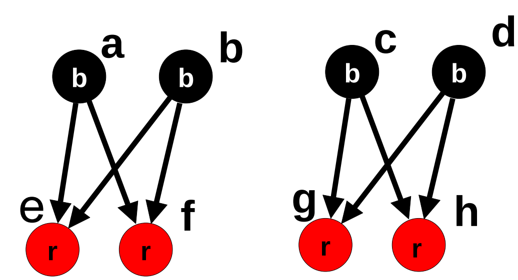
- Node u is automorphically equivalent to node v if there exists an automorphism p such that u = p(v)



Advantages of AE

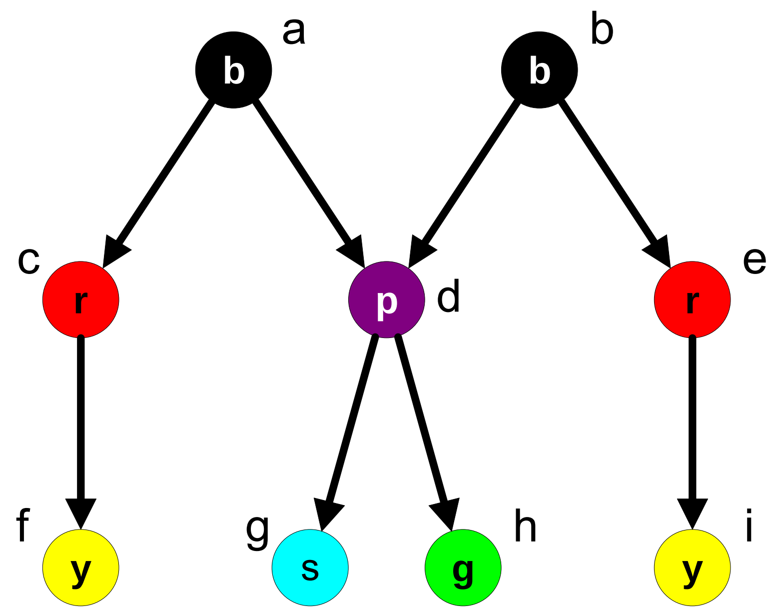
- Powerful, fundamental intuitive concept
- Truly structural/positional, not confounded by contiguity
- Captures results of exchange experiments
- Captures essentials of the role concept
- Generalization of structural equivalence that works with disconnected graphs





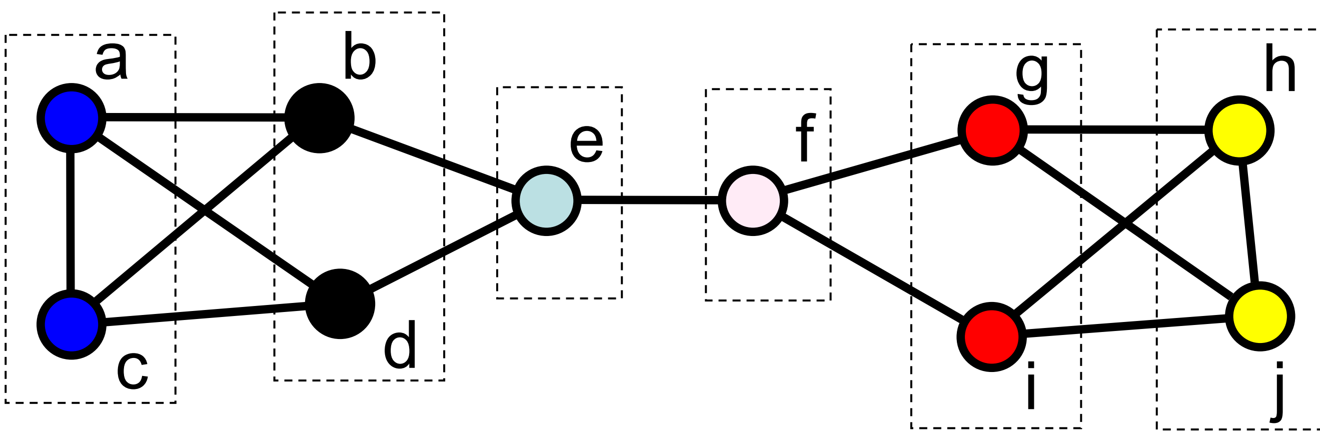
Problems with
Automorphic Equivalence

- A parent with 2 children does not play the same role as one with 3 children
- Extremely difficult to compute
- No obvious way to relax the concept for application to real world data
  - No two nodes are ever AE



Weak Structural Equivalence

- A coloration C of G(V,E) is weakly structural if C(u)=C(v) iff the permutation p=(u v) is an automorphism of G



(maximal) Regular Equivalence

The Dream

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- Society as concrete network of relationships among individuals
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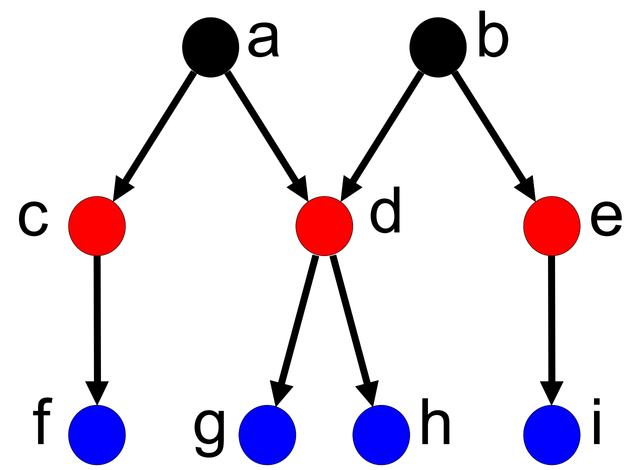
- Role freed from essentialist and culturalist definitions and defined in terms of characteristic relations among incumbents of positions, often reciprocally defined
  - Like functional role of species in ecosystem

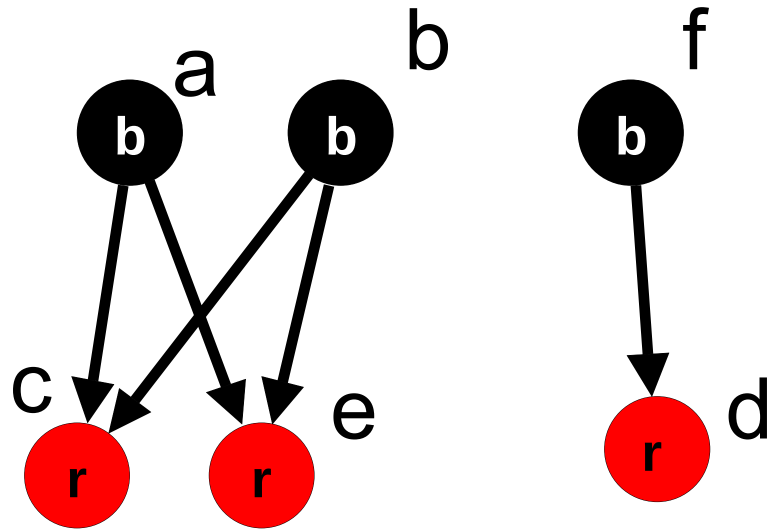
Regular Equivalence

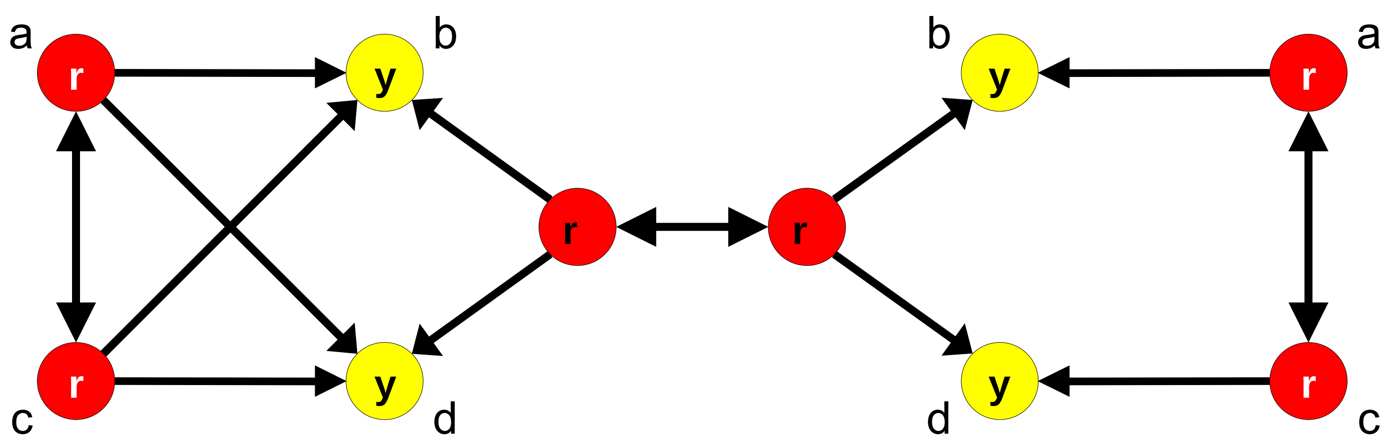
- Two nodes u and v are regularly equivalent if
  - Whenever u→c, there exists a node d such that v→d and c and d are regularly equivalent, and
  - Whenever c→u, there exists a node d such that d→v and c and d are regularly equivalent
- C(u)=C(v) implies C(N(u)) = C(N(v))
- Actually, C(u)=C(v) implies $C(N^{out}(u)) = C(N^{out}(v))$ and $C(N^{in}(u)) = C(N^{in}(v))$



Regularly equivalent nodes are not necessarily connected to the same third parties, but they are connected to equivalent third parties (though not necessarily in the same quantity)

Regular Equivalence





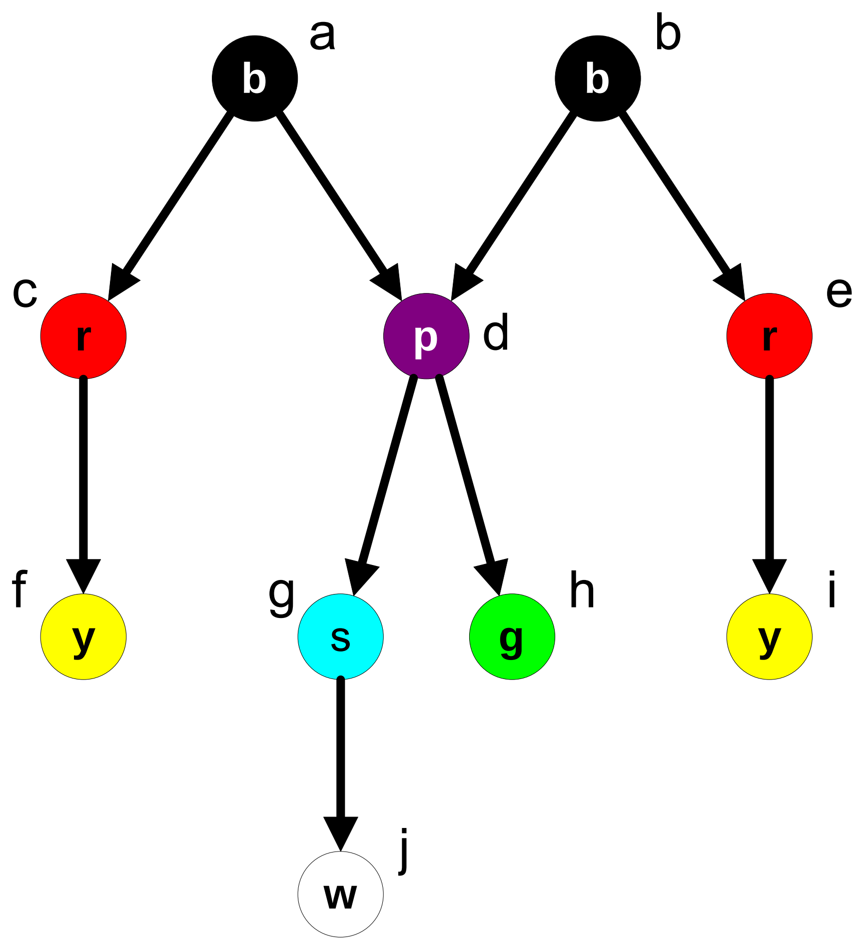
Regular Equivalence

- Captures role concept really well
  - Two actors are equivalent if they have the same relations with equivalent others
  - You call American airlines and talk to clerk about booking flight, while I call USAIR and do same with their clerk
    - You and I equivalent because the clerks are equivalent (and they are equivalent because you and I are equivalent)
- Less strict than automorphiic
  - Not concerned with quantities of colors
  - Finds more equivalent nodes

Regular Equivalence

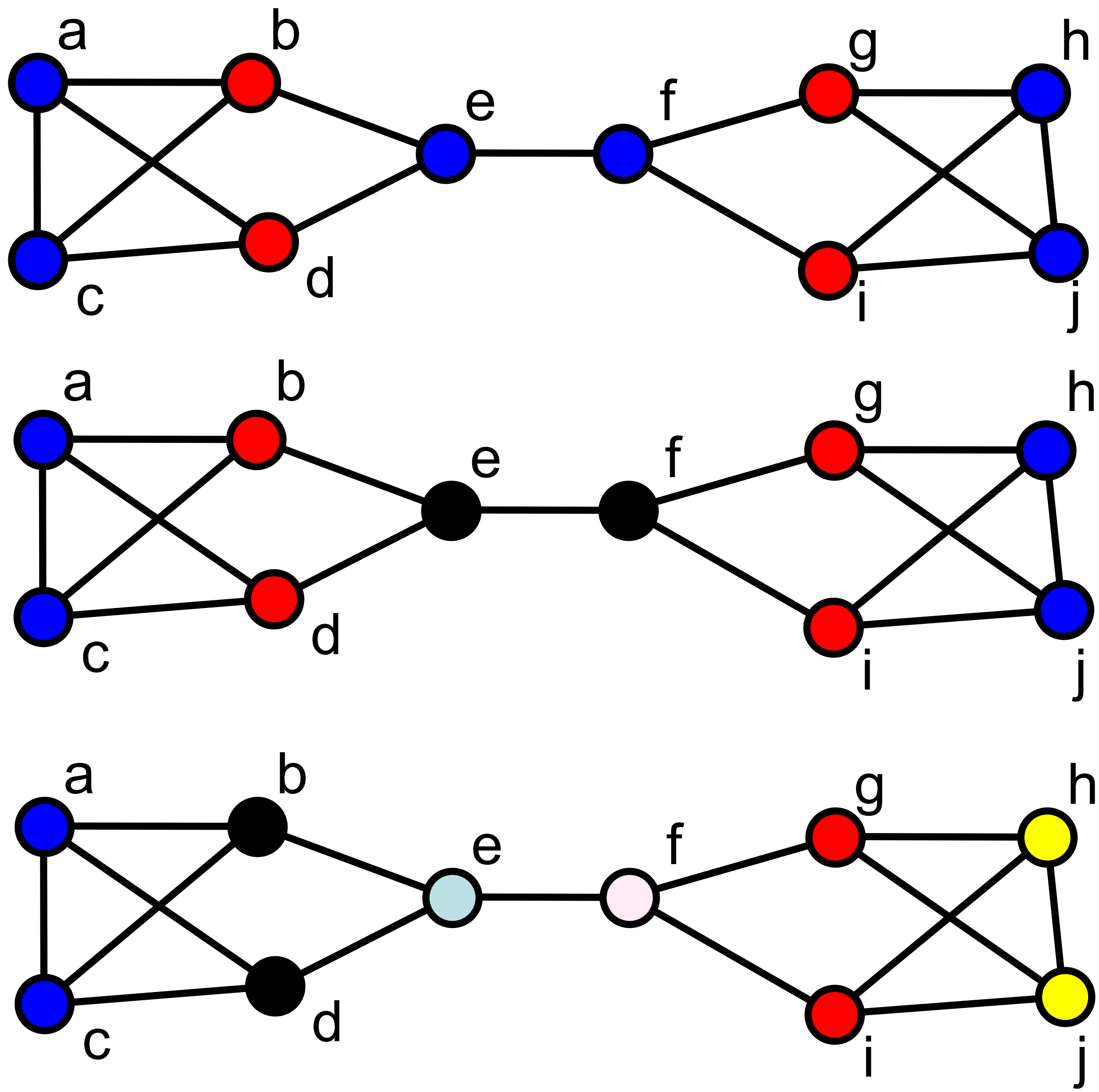
- Also captures position in hierarchies well
  - Including trophic group
- Relatively easy to compute (and to relax)
- Easy to generalize to 2-mode data
  - Consumers & brands
    - Segments & positions
    - identifying category boundaries
- Works well with multiple relations

Hierarchical Position



Problems with Regular Equivalence

- Often hard to interpret
  - Humans good at understanding pattern similarities, but not in the context of social ties
  - Data sets inappropriate for R.E. analysis
    - Too small, no real roles
- A graph may have multiple colorations that are regular – especially undirected graphs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A Family of Regular Equivs

- Every structural equivalence is also regular
- Automorphic is also regular*
- Actually form a lattice
- Somewhat like hierarchical clustering
  - Different levels of resolution

*At least as defined in this presentation. See JMS paper in 1994 for details.

Computation

- Relaxing concepts for real world data
- Two approaches
  - Discrete or blockmodel
    - Partition nodes into mutually exclusive classes such that departures from equivalence model are minimized
  - Profile similarity
    - For each pair of nodes, calculate the degree to which each pair is equivalent

Structural Equivalence

- Profile similarity method
  - Compute similarity/distance between rows of adjacency matrix
    - Correlation
    - Euclidean distance
  - Much argument over handling of diagonals
  - Can then MDS or cluster the resulting proximity matrix

Structural Equivalence

- Blockmodeling approach
  - Optimization method
  - Older Concor method
    - Actually based on profile method