Cohesive Subgroups

October 4, 2006

Based on slides by Steve Borgatti. Modifications (many sensible) by Rich DeJordy

Before We Start

• Any questions on cohesion?

Why do we care about Cohesive Subgroups

- 1. They exist!
- 2. They affect (social) processes we care about.
- 3. They offer the opportunity for data reduction:
 - Analyze separately
 - Aggregate cohesive subgroups

So, what are they?

 Many formalized definitions for lots of different flavors of cohesive subgroups, but, in general they are:

> Sections of the network in which actors are more closely relate to each other, on the whole, than to those outside that the group.

A Two By Two (and a half)

	Group by defined Algorithm	Group by defined Characteristics
Network/ Graph Theory	Newman-Girvan	Distance: Component, Clique, n-clique, n-clan, n-club Density: Clique, k-core, k-plex, Is-set, Iamba set
Proximity /Distance	Hierarchical Clustering MDS K-Means	(Core/Periphery) (Core/Periphery) (Combinatorial Optimization)

Groups defined by an algorithm based on distances/proximities

	Group by defined Algorithm	Group by defined Characteristics
Network/ Graph Theory	Newman-Girvan	Distance : Component, Clique, n-clique, n-clan, n-club
		Density : Clique, k-core,
		(Core/Periphery)
Proximity	Hierarchical Clustering	Factions
/Distance	MDS	(Core/Periphery)
	K-Means	
		(Combinatorial Optimization)

Johnson's Hierarchical Clustering

- Output is a set of nested partitions, starting with identity partition and ending with the complete partition
 - A "PARTITION" is a vector that associates each node with one and only one "group" (mutually exclusive)
- Different flavors based on how distance from a cluster to outside point/node is defined
 - Single linkage; connectedness; minimum
 - Complete linkage; diameter; maximum
 - Average, median, etc.



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BOS	0	206	429	1504	963	2976	3095	2979	1949
NY	206	0	233	1308	802	2815	2934	2786	1771
DC	429	233	0	1075	671	2684	2799	2631	1616
MIA	1504	1308	1075	0	1329	3273	3053	2687	2037
СНІ	963	802	671	1329	0	2013	2142	2054	996
SEA	2976	2815	2684	3273	2013	0	808	1131	1307
SF	3095	2934	2799	3053	2142	808	0	379	1235
LA	2979	2786	2631	2687	2054	1131	379	0	1059
DEN	1949	1771	1616	2037	996	1307	1235	1059	0

Closest distance is NY-BOS = 206, so merge these.



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	BOS N Y	DC	MIA	CHI	SEA	SF	LA	DEN
BOS/ NY	0	233	1308	802	2815	2934	2786	1771
DC	233	0	1075	671	2684	2799	2631	1616
MIA	1308	1075	0	1329	3273	3053	2687	2037
СНІ	802	671	1329	0	2013	2142	2054	996
SEA	2815	2684	3273	2013	0	808	1131	1307
SF	2934	2799	3053	2142	808	0	379	1235
LA	2786	2631	2687	2054	1131	379	0	1059
DEN	1771	1616	2037	996	1307	1235	1059	0

Closest pair is DC to BOSNY combo @ 233. So merge these.



M I X	B D A	a P	L X	8 0 8	N Y	Р С	C H I	D B N
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	DC	MIA	CHI	SEA	SF	LA	DEN
BOS/NY DC	0	1075	671	2684	2799	2631	1616
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	BOS/ NY/DC	MIA	СНІ	SEA	SF/LA	DEN
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	BOS/ NY/D C/ CHI	MIA	SEA	SF/L A	DEN
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BOS/NY/DC/ CHI	0	1075	2013	996
MIA	1075	0	2687	2037
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	BOS/ NY/D C/CHI		SF/LA
	/DEN	MIA	/SEA
BOS/NY/DC/ CHI/DEN	0	1075	1059
MIA	1075	0	2687
SF/LA/SEA	1059	2687	0



	BOS/ NY/D C/CH I/DE N/SF/ LA/S EA	MIA
BOS/NY/DC/CHI/DEN/SF/L A/SEA	0	1075
MIA	1075	0

Clustering Network Data

- Clustering requires symmetric data

 It will symmetrize if you don't
- Does better with a range of values



Average method helps...



Applying HiClus to Network Data

Geodesic Distances

• BETTER:

Compute geodesic distances first, then cluster the distance matrix

Or cluster the structural equivalence matrix

											1	1	1	1	1	1	1	1	1
		1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8
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		_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	_
1	HOLLY	0	4	2	1	1	2	2	2	1	2	4	1	3	1	2	3	4	3
2	BRAZEY	4	0	5	5	5	6	4	5	3	4	1	4	3	4	2	1	1	2
3	CAROL	2	5	0	1	1	2	1	2	3	4	5	3	2	3	3	4	4	3
4	PAM	1	5	1	0	2	1	1	1	2	3	5	2	2	2	3	4	4	3
5	PAT	1	5	1	2	0	1	1	2	2	3	5	2	2	2	3	4	4	3
б	JENNIE	2	6	2	1	1	0	2	1	3	4	6	3	3	3	4	5	5	4
7	PAULINE	2	4	1	1	1	2	0	1	3	4	4	3	1	3	2	3	3	2
8	ANN	2	5	2	1	2	1	1	0	3	4	5	3	2	3	3	4	4	3
9	MICHAEL	1	3	3	2	2	3	3	3	0	1	3	1	2	1	1	2	3	2
10	BILL	2	4	4	3	3	4	4	4	1	0	4	1	3	1	2	3	4	3
11	LEE	4	1	5	5	5	б	4	5	3	4	0	4	3	4	2	1	1	2
12	DON	1	4	3	2	2	3	3	3	1	1	4	0	3	1	2	3	4	3
13	JOHN	3	3	2	2	2	3	1	2	2	3	3	3	0	3	1	2	2	1
14	HARRY	1	4	3	2	2	3	3	3	1	1	4	1	3	0	2	3	4	3
15	GERY	2	2	3	3	3	4	2	3	1	2	2	2	1	2	0	1	2	1
16	STEVE	3	1	4	4	4	5	3	4	2	3	1	3	2	3	1	0	1	1
17	BERT	4	1	4	4	4	5	3	4	3	4	1	4	2	4	2	1	0	1
18	RUSS	3	2	3	3	3	4	2	3	2	3	2	3	1	3	1	1	1	0

Hierarchical Clustering



MDS

- Goal: Position nodes in space with "similar" nodes near each other
 - Nodes connected to the same people are "similar"
- Works better with more discriminating data (range of values)



K-Means

- Given a set of points in space
 - Place K new points, called centroids, (as far away from each other as possible) among the data
 - Associated each point with the closest centroid
 - Now, based on which points are associated with each centroid, reposition the centroid to the "middle" of all of them
 - Repeat until recentering does not move the centroids.
- Basically, groups are "clustered" based on their spatial relation to each other and these centroids.

Groups defined by an algorithm based on graph theoretic properties

	Group by defined Algorithm	Group by defined Characteristics
Network/ Graph Theory	Newman-Girvan	Distance: Component, Clique, n-clique, n-clan, n-club Density: Clique, k-core, k- plex, ls-set, lamba set
		(Core/Periphery)
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Newman-Girvan

- Calculate edge-betweenness for graph
- Remove edge with highest edge betweenness
- If number of components increases, create partition
- Recalculate edge betweenness & repeat until all nodes are isolates or maximum number of clusters reached/exceeded

Campnet Example Newman-Girvan 2 Cluster Partition

Where do you think the 2 Cluster Partition was?



Groups w/specified characteristics, based on Graph Theoretic Measures

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Components

- Maximally **connected** subgraph
 - In digraph there are strong and weak components:
 - Strong components mean everyone can reach everyone else, even when considering the one-way streets in the network
 - Weak components means, if we ignore the directionality of the ties, everyone is reachable by everyone else
 - A single weak component my comprise multiple strong components (pseudo-hierarchical, 2-levels)

Clique

- A maximal complete subgraph
 - Everyone is adjacent to everyone else
 - Distance & Diameter is 1
 - Density is 1
- Limitations
 - Undirected
 - 3+ nodes



Problems with Cliques

- Can be too many or too few
- If too many:
 - Can put minimum on size
 - Can look at overlap
- If too few, relax requirements in terms of
 - Distance:
 - n-cliques, n-clans, n-clubs
 - Density
 - k-cores, k-plexes, ls-sets, lamba sets

If too many....

- Look at CliqueSets
 Or CliqueOverlap
 - 2-mode matrix in GLA
- - Square matrix in MDS







Too Few, RELAX (Don't Do It) **Distance Requirement**

- n-Clique
 - Maximal subset with all nodes within n steps of each other
 - Path can include nodes not in n-Clique
 - A Clique is a 1-Clique (we don't count self-loops)

Is this a 2-Clique? NO! What about now?

But so is this!!!



Some are counter-intuitive (And not necessarily cohesive)



This is a 2-Clique



Red Nodes form a 2-Clique, so do Blues

So, we can force more cohesion

- n-Clan is an n-Clique whose diameter in the subgraph induced from the nodes in the n-Clique is <= n
 - Don't allow paths to go outside subset
- SUBGRAPH
 - A set of ties, together with ties among them
- An INDUCED SUBGRAPH
 - A subgraph defined by a set of nodes (or lines) and ALL the incident ties (or nodes)

2-Cliques vs. 2-Clans



{1,3,5} & {2,4,6} are

2

3

This is a 2-Clique & a 2-Clan

2-Cliques but not 2-Clans

Clans can be overlapping => $\{1,2,3\}, \{2,3,4\}, \{3,4,5\}, \{4,5,6\}, \{5,6,1\} \& \{6,1,2\} are$ 2-Clans and 2-Cliques

6

But, n-Clans have issues, too

- The n-Clique requirement is restrictive, so there are few found in the data
- Is {a,b,c,f} a 2-Clan?
- How many 2-Clans are there in this graph?



Loosening the restriction

- n-Clubs are, effectively, n-Clans that do not have the n-Clique requirement, or...
 - A maximal subset S such that the graph induced by the nodes S has a diameter <= n
 - Now {a,b,c,f} is a 2-Club, so is {a,b,e,f}



- Properties:
 - Painful (impossible) to compute
 - More plentiful than n-Clans
 - Overlapping

Another approach

- n-Cliques, n-Clans, and n-Clubs all start from the definition of Cliques and relax the distance requirement (all distances = 1) in varying ways
- But, Cliques also have maximum density (d = 1), and we can relax that definition instead...
- But for this, we must define the alpha operator, α, such that α(u,G) is the number of lines from node u to nodes in graph G

Relaxing the Density Requirement

- k-Plex
 - A clique where members don't have to be connected to everyone else, just all but k members, or...
 - a [maximal] subset S s.t. for all u in S, $\alpha(u,S) >= |S|-k$, where |S| is size of set S
 - All subsets of k-plexes are k-plexes (if non-maximal)
 - Get distance for free based on S, k.
 If k < (|S|+2)/2 then diameter <= 2
 - Numerous & Overlapping
 - May be more intuitive than distance-based measures
 - A Clique is a 1-plex (missing itself)

K-Plex



Is {a,b,d,e} a 2-plex? Is {a,b,c,d,e} a 2-plex? Is {a,b,d} a 2-plex?



Is the graph as a whole a 2-plex? Is it a 3-plex?

k-Core

- Sort of opposite approach from k-plex
 - Because the size of the group is not taken into account, k-cores are more directly about specifying how many ties MUST be present independent of how many nodes are in the core, whereas the k-plex is about both.
- A k-Core is maximal subgraph within which all nodes have ties to at least k other nodes
 - All nodes in a components are at least 1-Cores
 - Each nodes is assigned a "core" which is the largest k-core to which it belongs (and it therefore also belongs to all lower cores that exist)
 - K-cores are hierarchical and form a partition

Another definition

 A k-core is a maximal subgraph such that for all u in S, α(u,S) >= k



- All nodes are 2-core (and 1-core) Red nodes are 3-core.
- Great for analyzing large networks

LS-Sets

- Definition
 - Given a graph G(V,E), let H be a subset of V, and let
 K be <u>any</u> proper subset of H
 - H is **LS** if α (K,H-K) > α (K,V-H) for all K
 - All subsets of the LS set are more connected to other LS members than outsiders of LS set

or...

- H is **LS** if $\alpha(K) > \alpha(H)$, where $\alpha(K) \Rightarrow \alpha(K,G-K)$
 - Subsets better off joining LS set
 - This one's usually easier to compute







- H is LS if α (K,H-K) > α (K,V-H)
 - Use when K is large

or ...

• H is LS if $\alpha(K) > \alpha(H)$

- Use when K is small



LS-Sets

- Properties very cohesive
 - Wholly nested or disjoint
 - No partial overlaps
 - More ties within than between
 - Everyone more connected inside than outside
 - Contain no minimum weight cutsets
 - lie on either side of "fault lines"
 - Multiple edge-independent paths within
 - High edge-connectivity

Lambda Sets

- Definition
 - A set of nodes S is a lambda set if, for all, *a*, *b*, *c* in S and *d* not in S, $\lambda(a,b) > \lambda(c,d)$
 - where $\lambda(u,v)$ is the number of edge-independent paths from node u to node v, which is also the minimum number of ties that must be removed in order to disconnect u and v
 - Members of Lamba Sets have more independent paths to ALL other group members than ANY outsider
- Properties
 - Robust
 - very difficult to disconnect even with intelligent attack
 - Mutually exclusive or wholly inclusive
 - no partially overlapping groups
 - Pure
 - defined on a single attribute (edge connectivity)

Lambda Sets



{1,2,3,4} {1,2,3,4,5,6,7,8} {9,10,11,12} Non-Trivial Lambda Sets {1,2,3,4} {1,2,3,4,5,6,7,8} {9,10,11,12} {5,6,7,8}

Groups w/specified characteristics, based on Proximities

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	K-Means			
		(Combinatorial Optimization)		

Factions

- Computationally arrange nodes into mutually exclusive groups such that some predefined criteria is optimized
 - For example, make groups that maximize density of internal ties and minimize density of external ties

Campnet Example

Group Assignments:



Core-Periphery Models

- A core periphery structure has a single cohesive subgroup with a set of other nodes, loosely connected to the core
- Core members interact with (lots of) other core members
- Peripheral members interact with (a few) core members

Finding Core/Periphery Structures

- Two ways to deal with it...
 - One is a special case of factions, which maximizes density of core-to-core relations and minimizes all others (categorical)
 - Another is a continuous model that calculates a "coreness" which is how much this node looks like a core node (continuous)